Tutorial & Review

Similarity and scaling networks for low-pressure discharge plasmas

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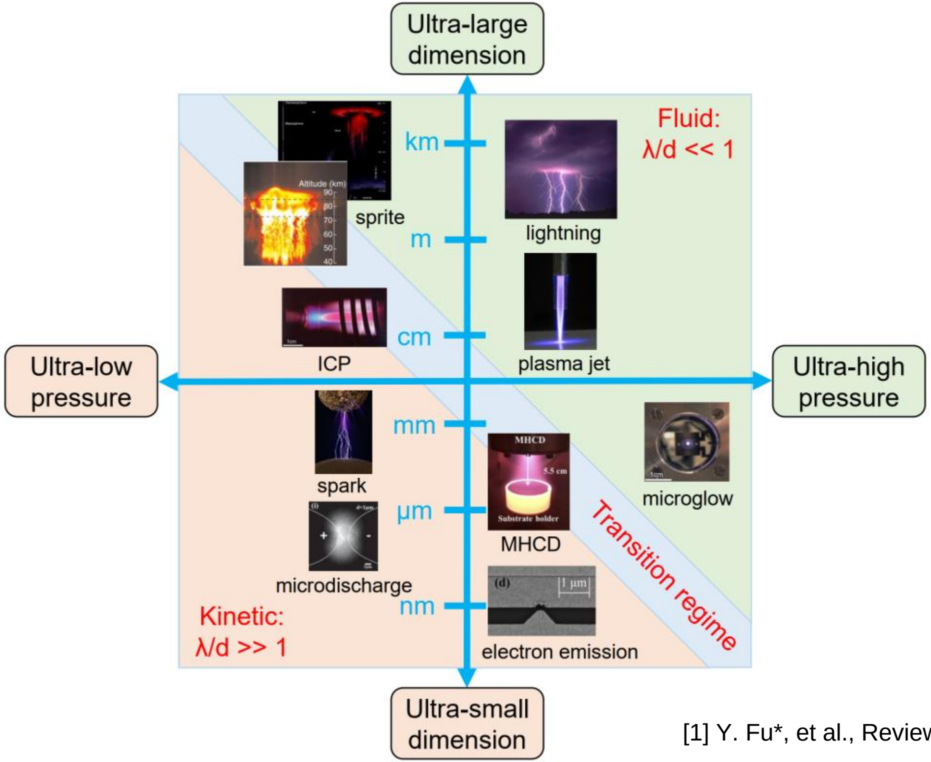
Outline

Plasma scales

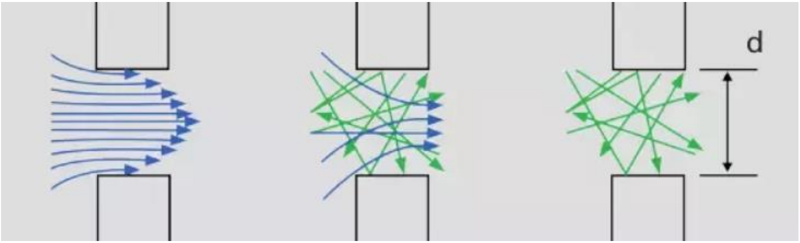
- **Characteristic length of plasmas**



Knudsen number: $\mathrm{Kn} = \lambda/d$

(1) Fluid (local) $\lambda/d \ll 1$

(2) Particle (nonlocal) $\lambda/d \gg 1$



Fluid **Hybrid** **Particle**

[1] Y. Fu*, et al., Reviews of Modern Plasma Physics **7**, pp.1-81 (2023).

Townsend mechanism (1/2)

- **Upscaling & downscaling**

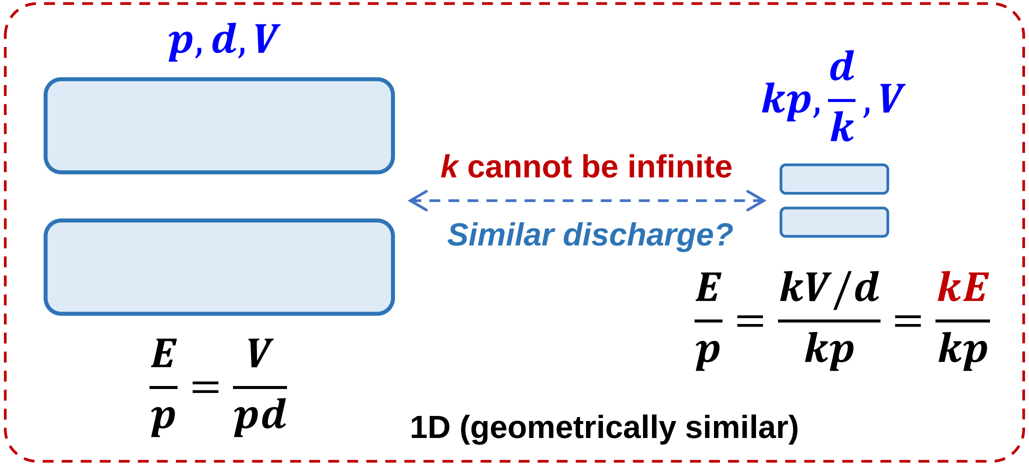
$$\frac{\alpha}{p} = A \cdot \exp(-Bp/E)$$

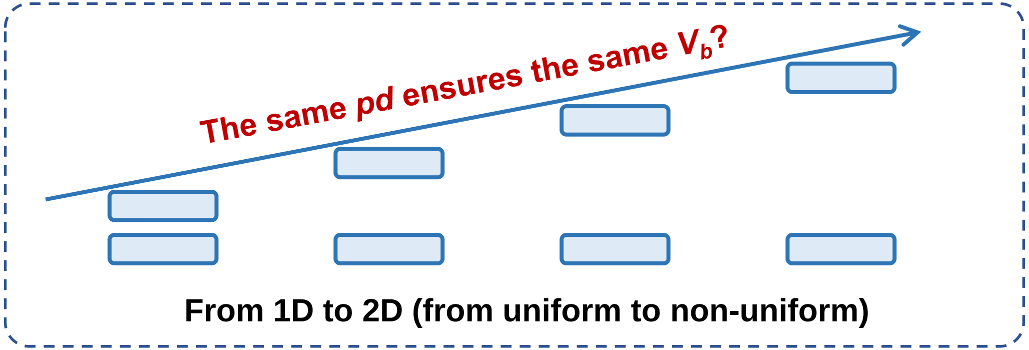
$$\gamma_{\rm se}[\exp(\alpha d) - 1] = 1$$

$$U_{\rm b} = \frac{Bpd}{\ln(Apd) - \ln[\ln(1+1/\gamma_{\rm se})]}$$

$$j_\alpha = e n_o e^{\alpha d} = j_o e^{\alpha d}$$

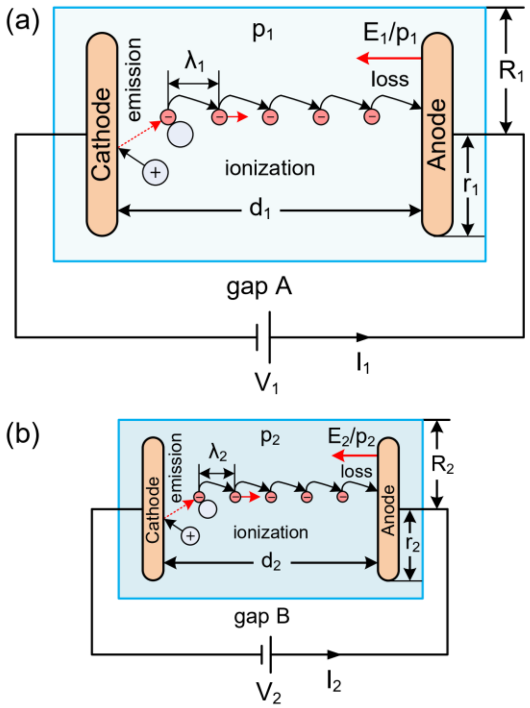
The same pd, the same breakdown voltage and the same current!

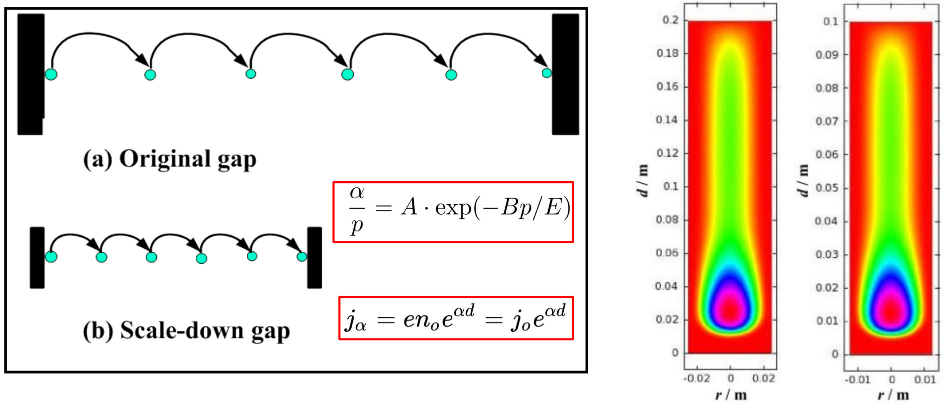




Townsend mechanism (2/2)

- **Geometrically similar discharge systems**





(p_1, d_1) ⟷ (p_2, d_2)

$$d_1/\lambda_1 \sim p_1 d_1 = p_2 d_2 \sim d_2/\lambda_2$$

$$E_1\lambda_1 \sim E_1/p_1 = E_2/p_2 \sim E_2\lambda_2$$

Similar discharge vessels

[1] Y. Fu*, et al., Reviews of Modern Plasma Physics **7**, pp.1-81 (2023).

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Motivation

Question: Can the discharge processes be replicated from one system to another?

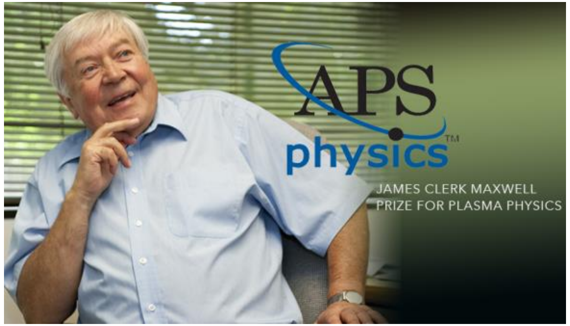
Scaling laws for dynamical plasma phenomena

D. D. Ryutov[a),b)]
Lawrence Livermore National Laboratory, Livermore, California 94550, USA

(Received 30 May 2018; accepted 14 September 2018; published online 17 October 2018)

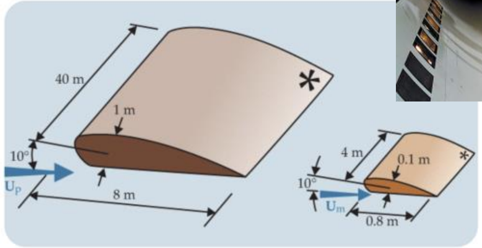
A scaling and similarity technique is a useful tool for developing and testing reduced models of complex phenomena, including plasma phenomena. In this paper, similarity and scaling arguments will be applied to highly dynamical systems where the plasma is evolving from some initial to some final state, which may differ dramatically from each other in size and plasma parameters. A question then arises whether, in order to better understand the behavior of one such system, is it possible to create another system, possibly much smaller (or larger) than the original one, but whose evolution would accurately replicate that of the original one, from its initial to its final state. This would allow a researcher, by an experimental study of this second system, to make confident predictions about the behavior of the first one (which may be otherwise inaccessible, as is the case of some astrophysical objects, or too expensive and hard to diagnose, as in the case of fusion applications of pulsed plasma systems, or for other reasons). The scaling and similarity techniques for dynamical plasma systems will be presented as a set of case studies of problems from various domains of plasma physics, including collisional and collisionless plasmas. Among the results discussed are similar for MHD systems with an emphasis on high-energy-density laboratory astrophysics, interference between collisionless and collisional phenomena in the context of shock physics, and similarity for liner-imploded plasmas. *Published by AIP Publishing.*

https://doi.org/10.1063/1.5042254



D. D. Ryutov (LLNL)

Fluid mechanics

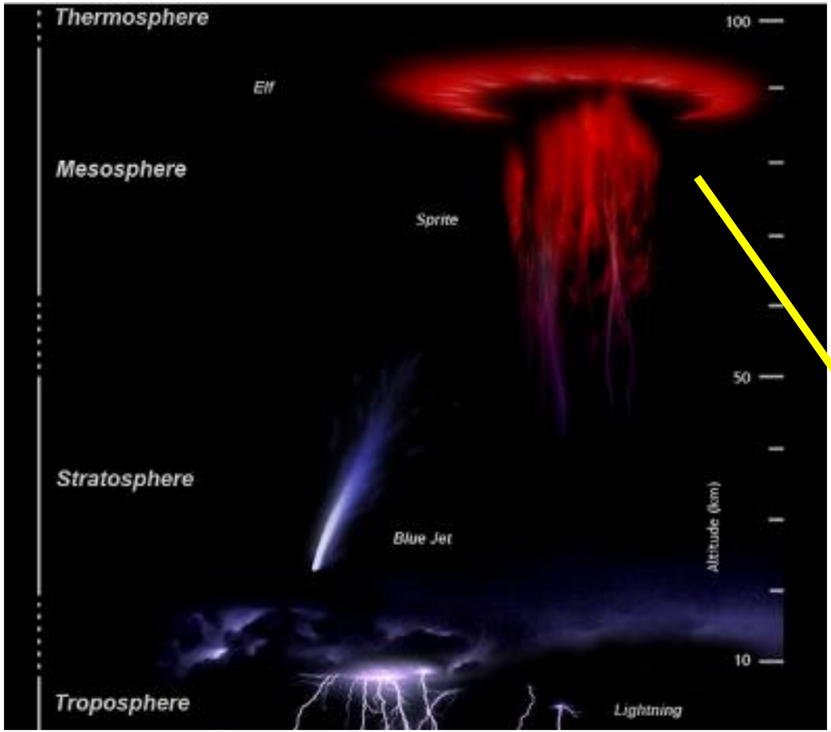


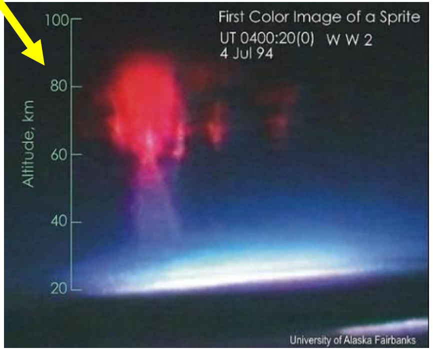
Scaled aircraft wind tunnel

[1] D. D. Ryutov, Phys. Plasmas **25**, 100501 (2018).

Ultralarge-scale discharges (1/3)

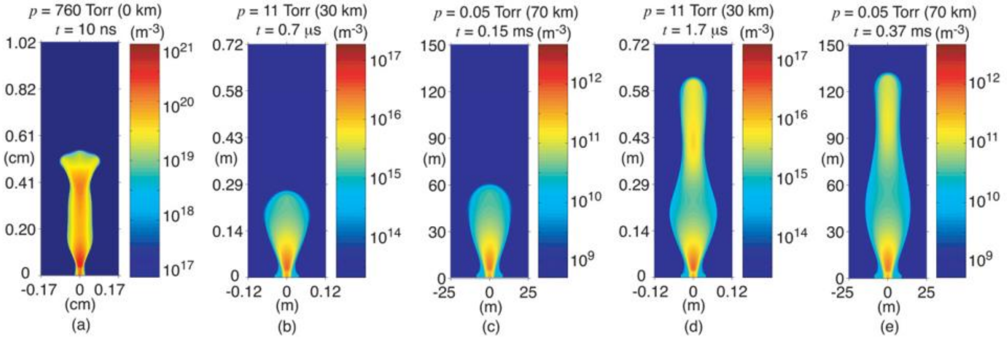
- **Large-scale discharges in mesosphere**





First color image of the Sprite

Similarity in streamers



INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS D: APPLIED PHYSICS
J. Phys. D: Appl. Phys. 39 (2006) 327–334 doi:10.1088/0022-3727/39/2/013

Effects of photoionization on similarity properties of streamers at various pressures in air

N Liu and V P Pasko

Communications and Space Sciences Laboratory, Department of Electrical Engineering, Pennsylvania State University, University Park, PA 16802, USA

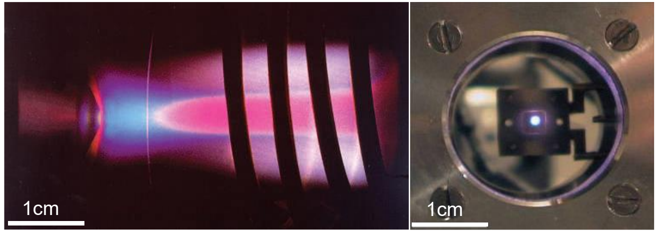
Local field approximations

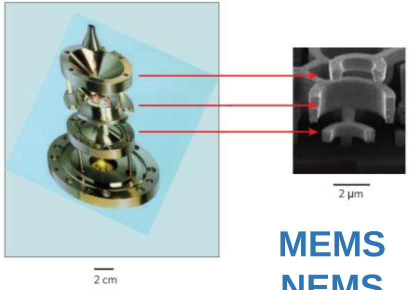
Fluid model

[1] N. Liu, et al. JPD, 2006.
[2] V. P. Pasko, et al. PSST, 2007.
[3] D. D. Sentman, GRL, 1995.
[4] R. C. Franz, et al. Science, 1990.

Ultrasmall-scale discharges (2/3)

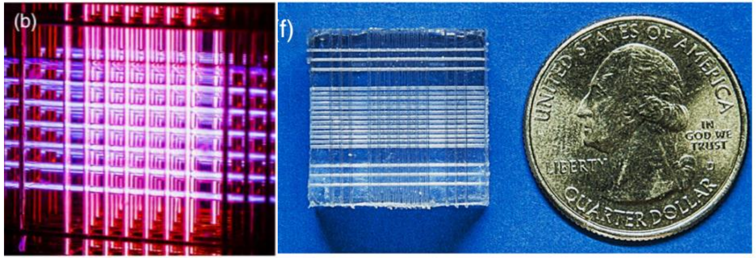
- **Miniaturized plasma devices**





MEMS
NEMS

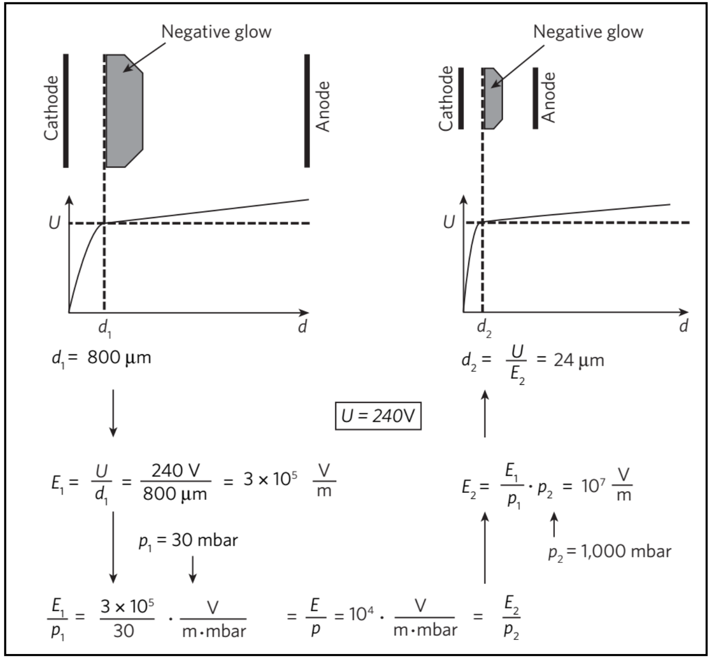
Conventional ICP (1kW) and MHCD (<1W)



Microplasma 3D array

P. P. Sun, J. G. Eden, et al., Appl. Phys. Rev. 6, 041406 (2019)

Similar microglow discharges



D. Janasek, et al. Nature 442, 374 (2006)

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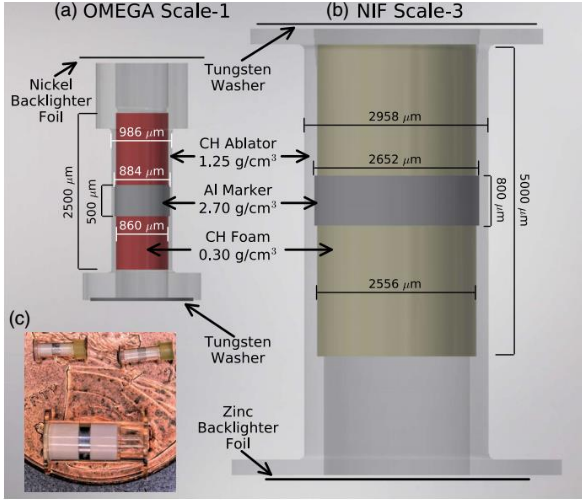
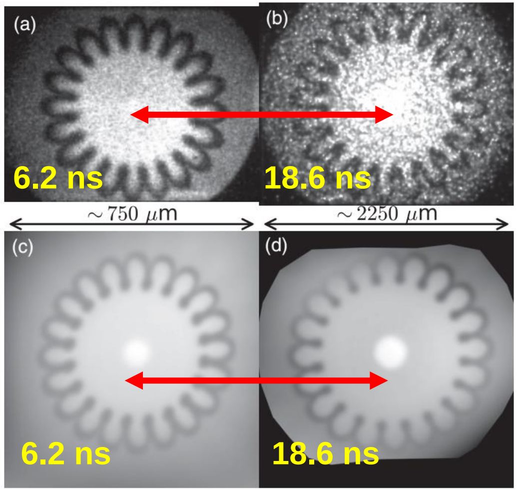
Laser-driven implosion (3/3)

- **Hydrodynamic scaling** $\boxed{\mathbf{x} \rightarrow \lambda\mathbf{x} \text{ and } t \rightarrow \lambda t}$



3 times enlarged in dimension

(a) OMEGA shot 93069 and (b) NIF shot N190212-003



experimental results

simulation results

3 times enlarged in time

Scale-invariant Rayleigh-Taylor instability growth in laser-driven plasmas enables detailed cross comparisons between targets of different dimensional scales.

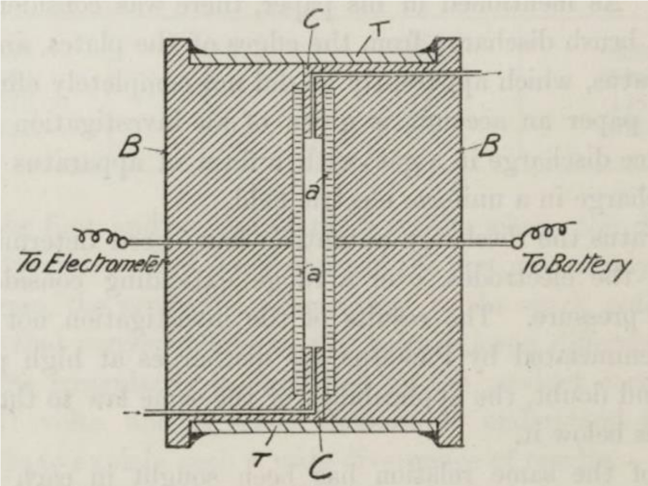
[1] J. P. Sauppe, et al., Phys. Rev. Lett. **124**, 185003 (2020).

Outline

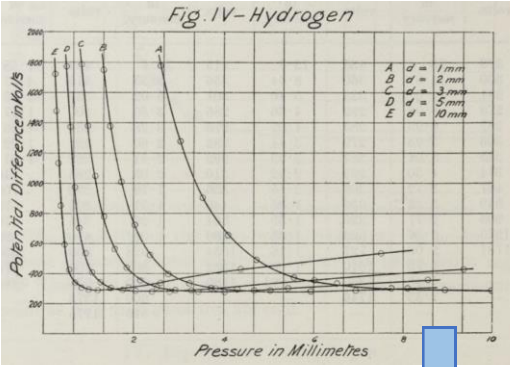
1. Introduction
2. Historical development (Breakdown similarity)
3. Mathematical derivation (F- and B-similarity)
4. Similarity in discharge plasmas (RF plasmas)
5. Summary

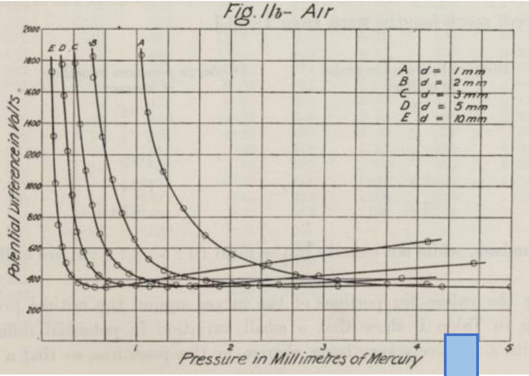
Historical development (1/3)

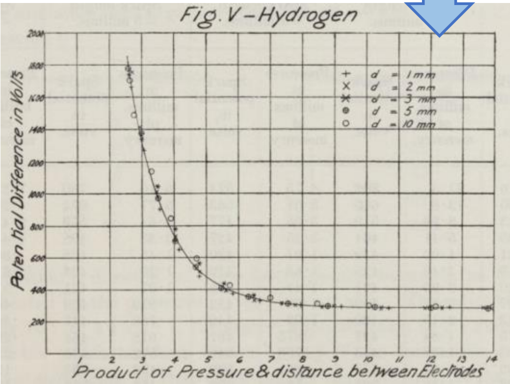
- **Similarity parameters for DC breakdown**

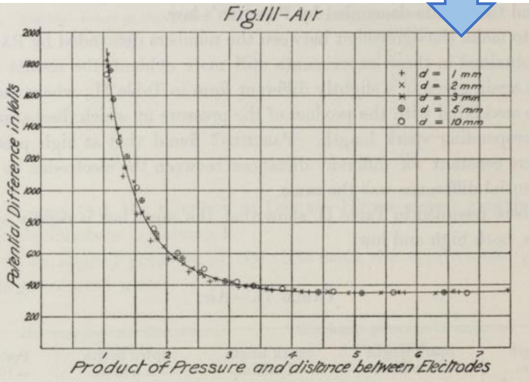


Left branch of Paschen's curve





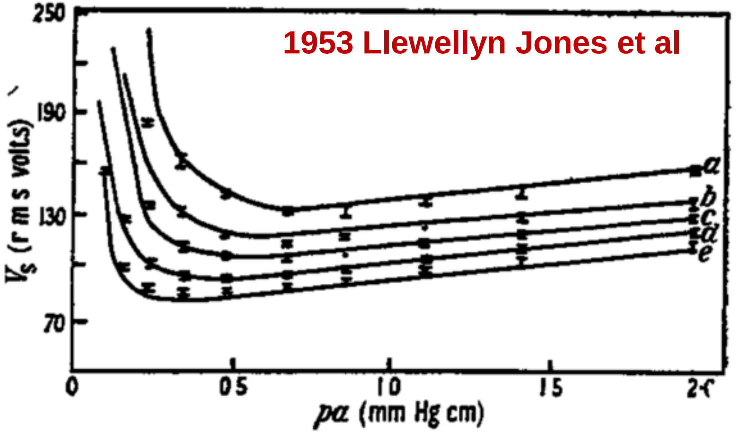




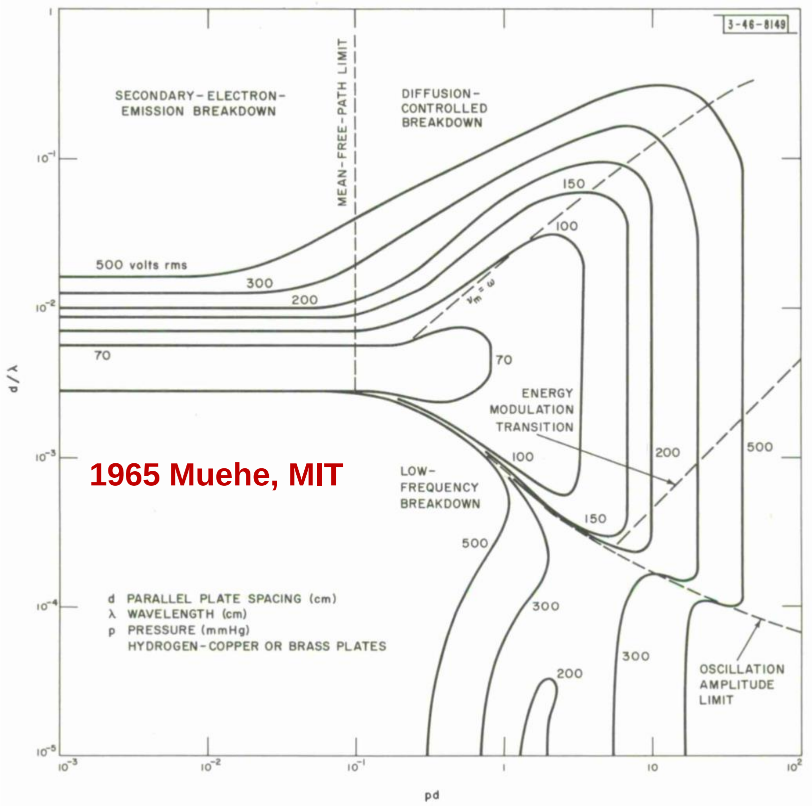
[1] W.R. Carr, X. On the laws governing electric discharges in gases at low pressures. Phil. Trans. R. Soc. Lond. A **71**, 374–376 (1903)

Historical development (2/3)

- **Similarity parameters for AC breakdown**



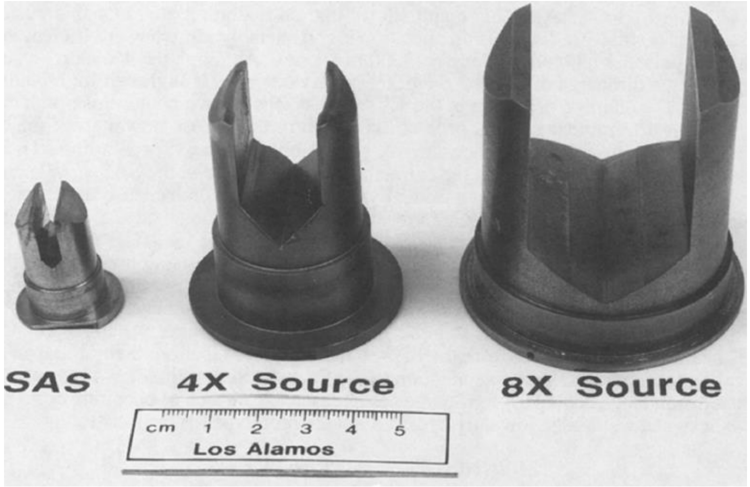
The line curves *a, b,* c, *d,* and e are those with tube C at frequencies *f* of 10, 20, 30, 50 and 70 Mc/s, and the **points marked** are actual measurements obtained with the geometrically similar tube B, of twice the linear dimensions, at half the pressure and at frequencies *f/2.*



[1] F. Llewellyn Jones and G. C. Williams, Proc. Phys. Soc. B **66** 17 (1953)
[2] C.E. Muehe, Ac breakdown in gases. MIT Lincoln Laboratory, Cambridge (1965)

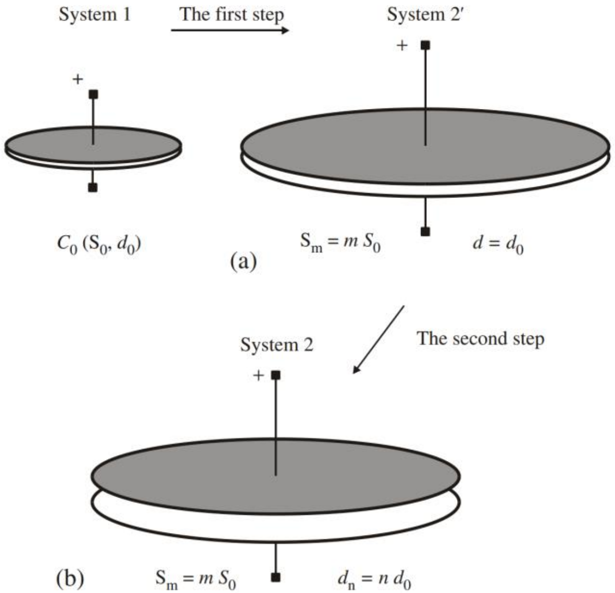
Historical development (3/3)

- **Similarity design of discharge devices**
  - ◻ enable more frequent and thorough analyses
  - ◻ less time-consuming to construct and test
  - ◻ less costly than the prototype



Penning Surface-Plasma Sources

H. Smith, et al., Rev. Sci. Instrum. **65**, 123-128 (1994)



P. Osmokrovic, et al., Plasma Sources Sci. Technol. **15**, 703–713 (2006)

Modified Paschen's law (1/3)

- **Breakdown similarity:**

$$\frac{\alpha}{p} = A \cdot \exp(-Bp/E) \qquad \gamma_{\text{se}}[\exp(\alpha d) - 1] = 1$$

$$U_{\text{b}} = \frac{Bpd}{\ln(Apd) - \ln[\ln(1 + 1/\gamma_{\text{se}})]}$$

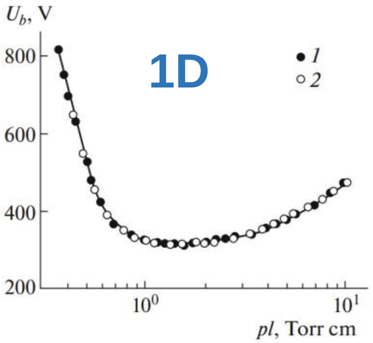
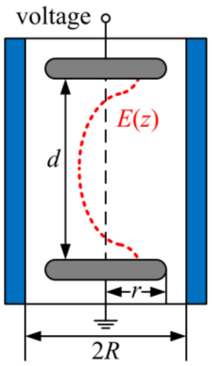
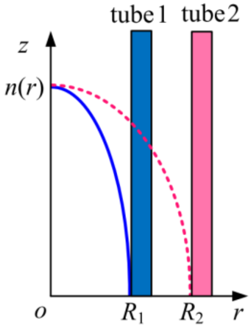
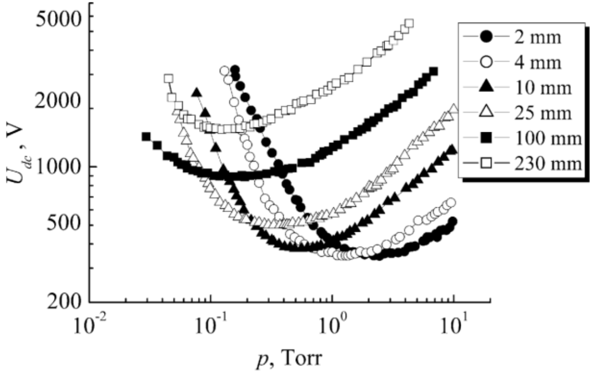
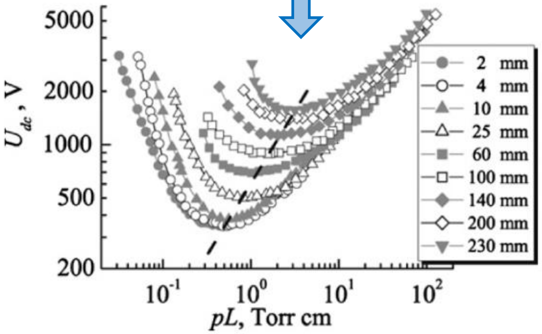


Fig. 14. Modified Paschen curve for argon at $l/r = 2.4$ [67]; $l =$ (*1*) 1.1 and (*2*) 3.3 cm.





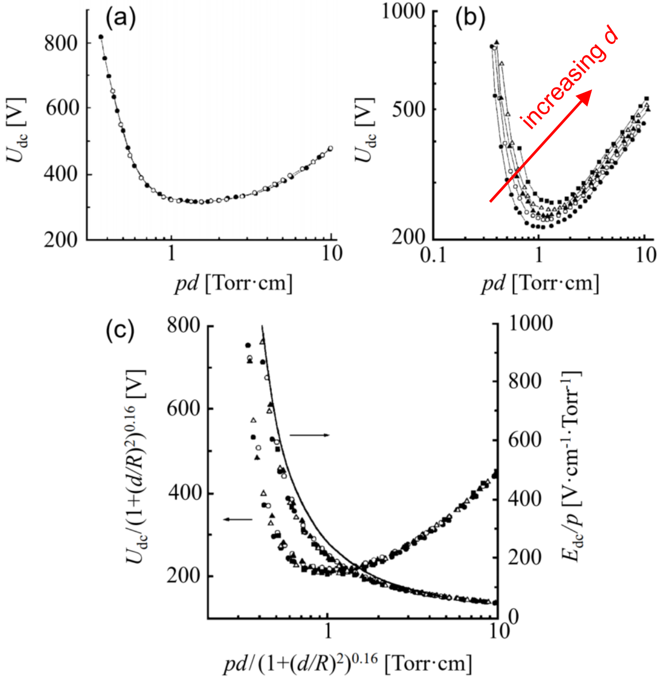




[1] V.A. Lisovskiy, et al., J. Phys. D Appl. Phys. **33**(21), 2722–2730 (2000)
[2] V.A. Lisovskiy, et al. Phys. Lett. A **375**(19), 1986–1989 (2011)
[3] V.A. Lisovskiy, et al., Vacuum **145**, 19–29 (2017)
[4] Y. Fu, et al., IEEE Trans. Plasma Sci. **47**, 1994 (2019)
[5] Y. Z. Ionikh, Plasma Phys. Rep. **46**, 1015–1044 (2020)

Modified Paschen's law (2/3)

- **Breakdown similarity:**



$$\frac{\alpha}{p} = A \cdot \exp(-Bp/E)$$

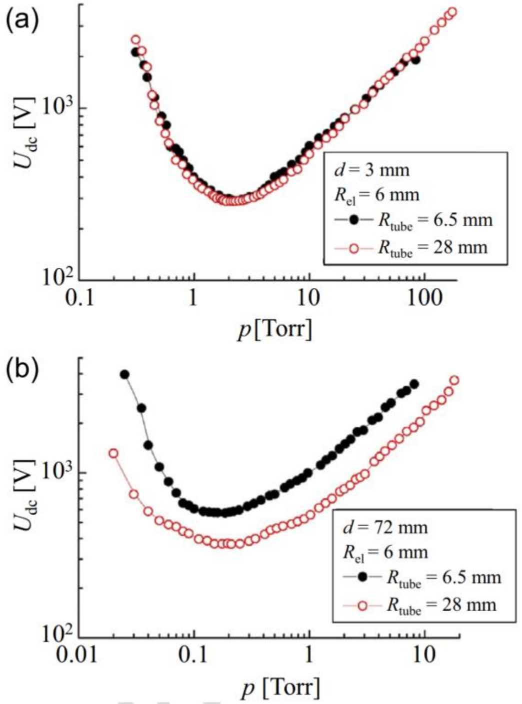
$$\alpha_{\mathrm{eff}} = \alpha - \frac{D_e}{v_e}\left(\frac{2.405}{R}\right)^2$$

$$U^*_{\mathrm{dc}} = \frac{U_{\mathrm{dc}}}{[1+(d/R)^2]^a},$$

$$(pd)^* = \frac{pd}{[1+(d/R)^2]^a},$$

for argon, $a \approx 0.16$.

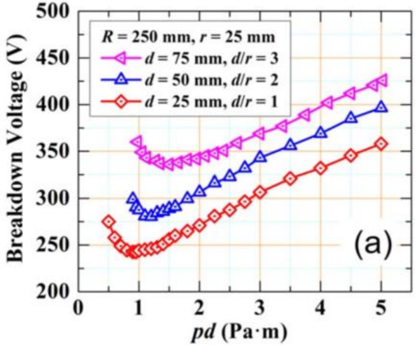
$$\boxed{U_{\mathrm{dc}} = f(pd, d/r, d/R),}$$

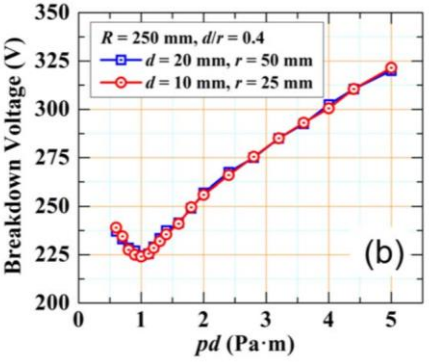


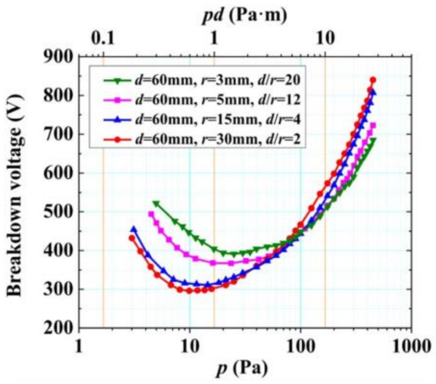
[1] V.A. Lisovskiy, et al., J. Phys. D Appl. Phys. **33**(21), 2722–2730 (2000)
[2] V.A. Lisovskiy, et al., Vacuum **145**, 19–29 (2017)

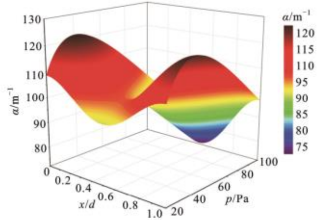
Modified Paschen's law (3/3)

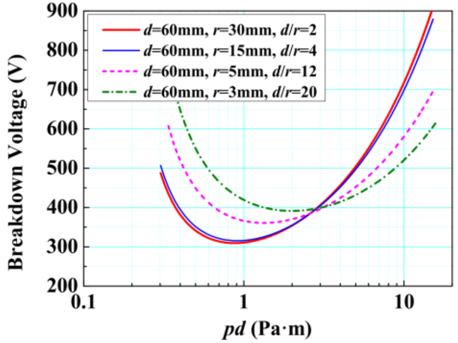
- **Breakdown similarity: dc**

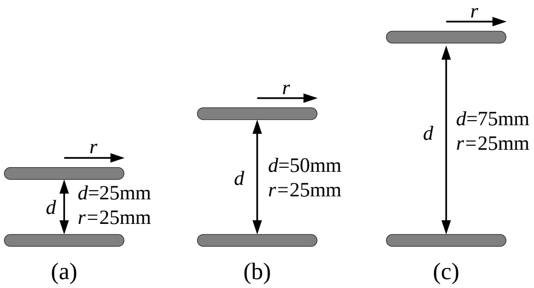












$$U_{\mathrm{dc}} = f(pd, d/r, d/R),$$

$$\boxed{d/R = d/R_{\mathrm{tube}} \to 0}$$

$$U_{\mathrm{dc}} = f(pd, d/r)\,|_{r \neq R},$$

$$\boxed{d/r = d/R_{\mathrm{el}} \to 0}$$

$$U_{\mathrm{dc}} = f(pd)$$

[1] Y. Fu, et al., High Volt. **1**, 86 (2016)
[2] Y. Fu, et al., Phys. Plasmas **23**, 093509 (2016)
[3] Y. Fu, et al., Phys. Plasmas **24**, 023508 (2017)

Breakdown similarity

- **Pulsed discharges:** $\boxed{U_\mathrm{b} = f(p, g) = f\left(kp, \frac{g}{k}\right)}$

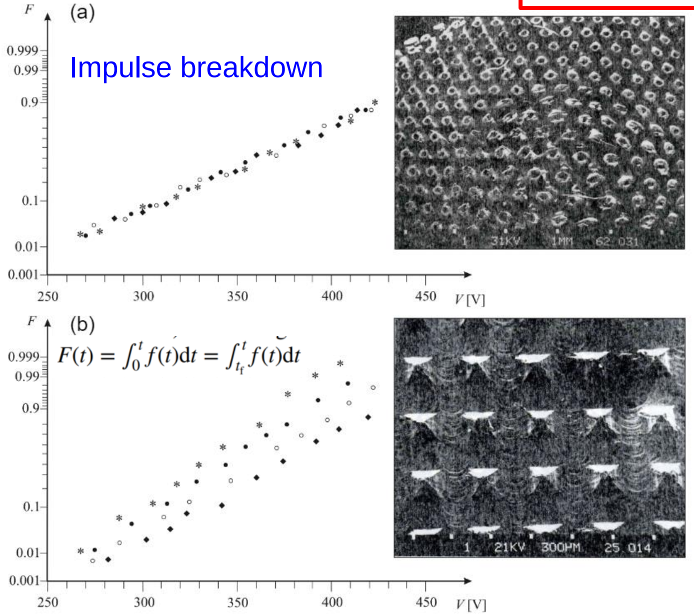
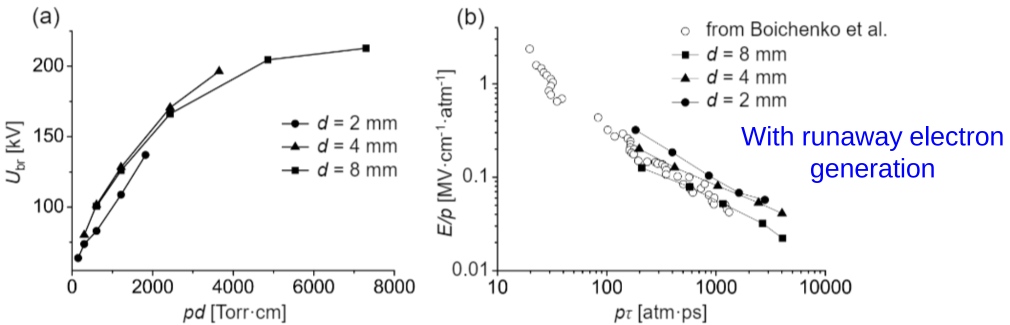
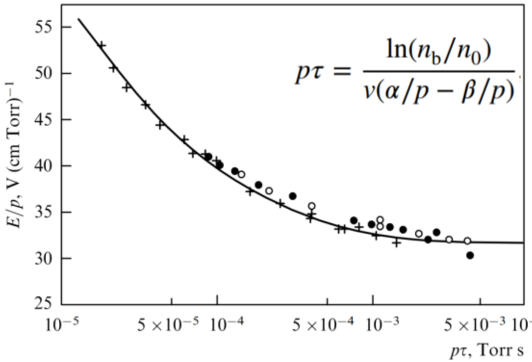


Fig. 13 Cumulative distribution of the impulse breakdown voltage in similar N_2-insulated systems with $pd = 0.05$ kPa·mm; $(p, d) = *$ (0.01 kPa, 5 mm), ○ (0.02 kPa, 2.5 mm), ● (0.025 kPa, 2 mm), and ♦ (0.05 kPa, 1 mm). (a) Electrodes with microscopic cavities; (b) electrodes with microscopic pyramids. Reproduced with the permission of IEEE Publishing from [213].

S. Dekić, et al. IEEE Trans. Dielectr. Electr. Insul. **17**(4), 1185–1195 (2010)

E.K. Baksht, et al., Russ. Phys. J. **60**(8), 1413–1418 (2017)

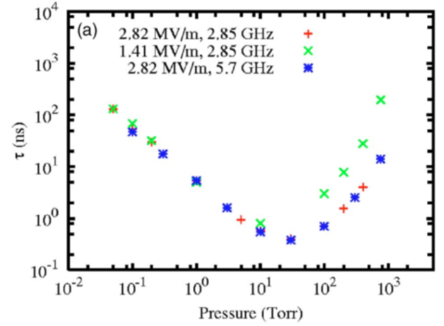


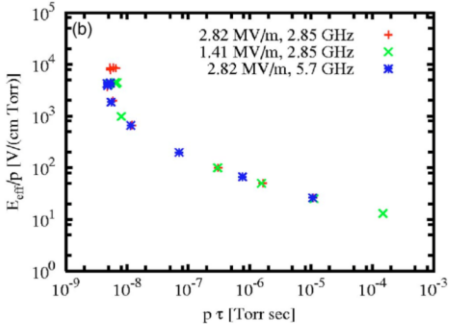


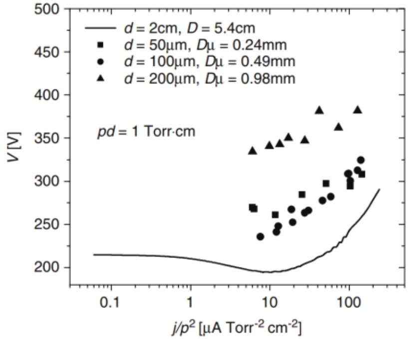
G.A. Mesyats, Similarity laws for pulsed gas discharges. Phys.-Usp. **49**(10), 1045–1065 (2006)

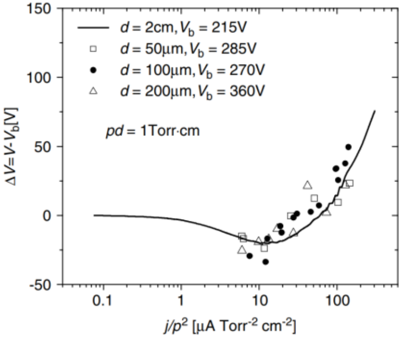
Breakdown similarity (1/2)

- **Microwave and microdischarge**

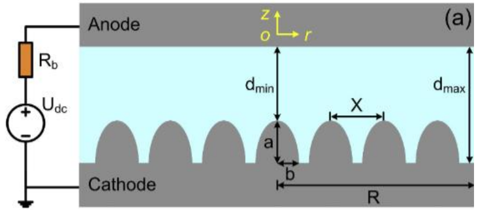




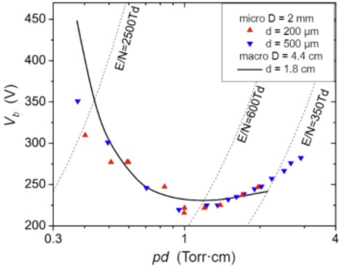




Nonplanar surface



Flat surface



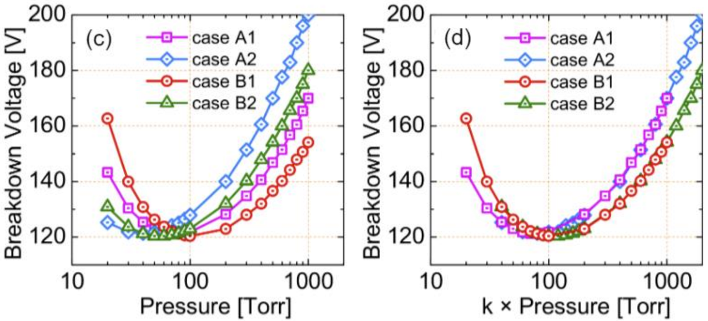
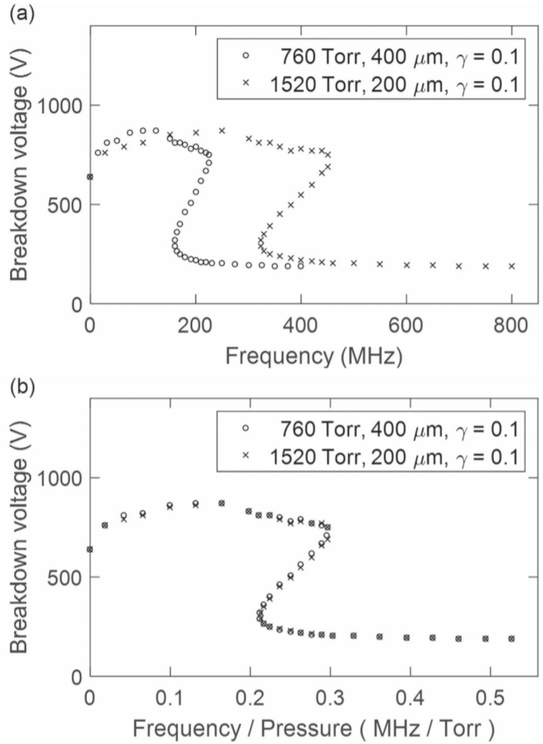


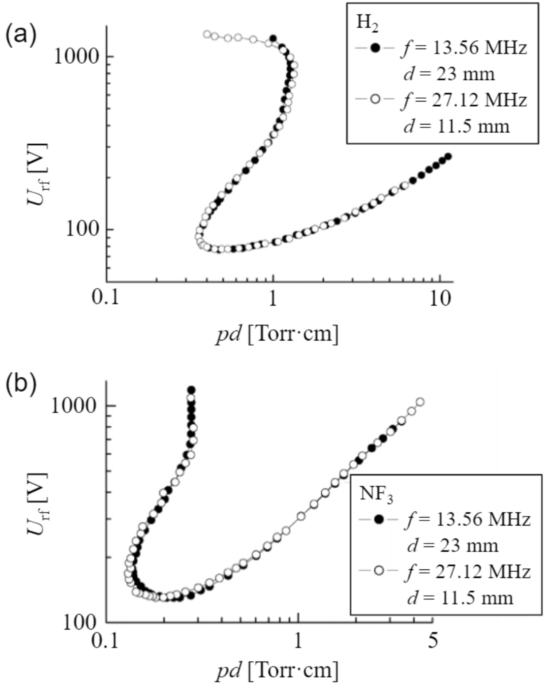
Fig. 15 (a) Schematic of a microgap with concentric protrusions on the cathode surface; (b) 3D view of the electrode with concentric protrusions; (c) breakdown curves as a function of gas pressure for geometrically similar gaps (A1 and A2, B1 and B2); and (d) breakdown curves as a function of the scaled gas pressure (scaling factor $k = 1$ and 2). Reproduced with the permission of AIP Publishing from [225].

[1] H. C. Kim et al., Phys. Plasmas **13**, 123506 (2006)
[2] Z. L. Petrovic, et al, J. Phys. D **41**, 194002 (2008)
[3] N. Škoro, J. Phys. Conf. Ser. **399**, 012017 (2012)
[4] Y. Fu, et al., Appl. Phys. Lett. **114**, 014102 (2019)

Breakdown similarity (2/2)

- **Radio-frequency breakdown:**





$$U_{\mathrm{rf}} = f(pd, f/p)$$

⇩

$$U_{\mathrm{rf}} = f(pd, fd)$$

⇩

$$U_{\mathrm{rf}} = f(pd, fd, d/R)$$

[1] M.U. Lee, et al., Plasma Sources Sci. Technol. **26**(3), 034003 (2017).
[2] V. Lisovskiy, et al., Similarity law for rf breakdown. EPL **82**(1), 15001 (2008).

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Violation of breakdown scaling (1/2)

- **DC Microgap breakdown:**

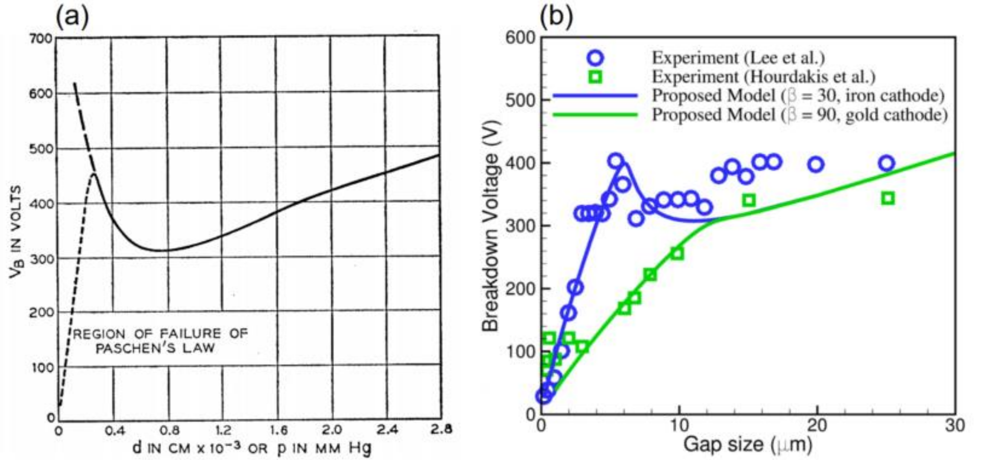
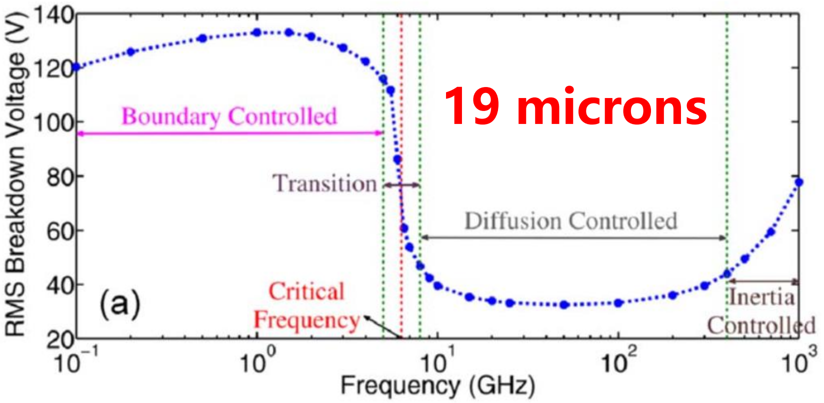


Fig. 17 (a) Breakdown voltage plotted against the gap distance at a constant pressure of 1 mm·Hg and against the gas pressure at a constant distance of 1 cm, which demonstrates the failure of Paschen's law at small distances. Reproduced with the permission of APS Publishing from [231]. (b) Numerical model proposed to predict the violation of Paschen's law incorporating electron field emission, which is also compared with experimental measurements obtained by Lee et al. [237] and Hourdakis et al. [238, 239]. Reproduced with the permission of AIP Publishing from [36].

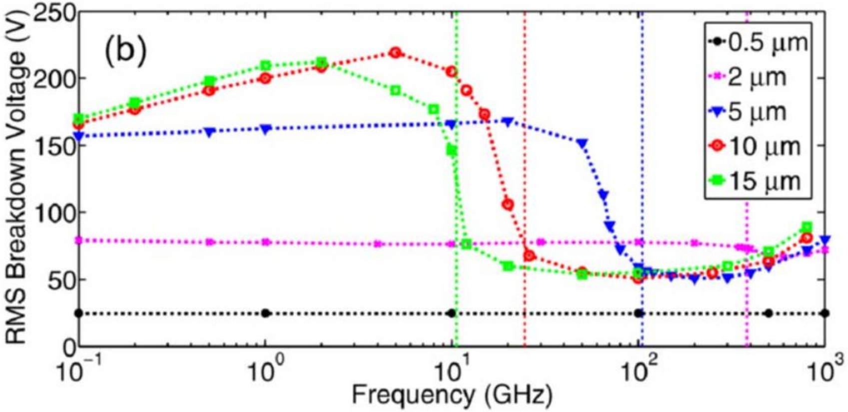
[1] W.S. Boyle, P. Kisliuk, Departure from Paschen's law of breakdown in gases. Phys. Rev. **97**(2), 255–259 (1955)
[2] A. Venkattraman, A.A. Alexeenko, Scaling law for direct current field emission-driven microscale gas breakdown. Phys. Plasmas **19**(12), 123515 (2012)

Violation of breakdown scaling (2/2)

- **HF microgap breakdown**



PIC simulation



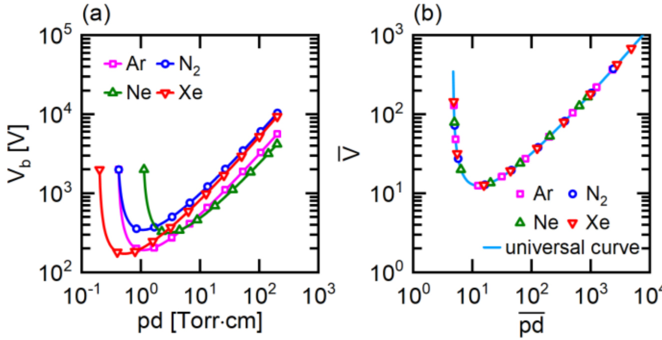
Without field electron emission

Transition to field emission regime

A. Semnani, A. Venkattraman, A. A. Alexeenko, and D. Peroulis, Appl. Phys. Lett. **103**, 063102 (2013)

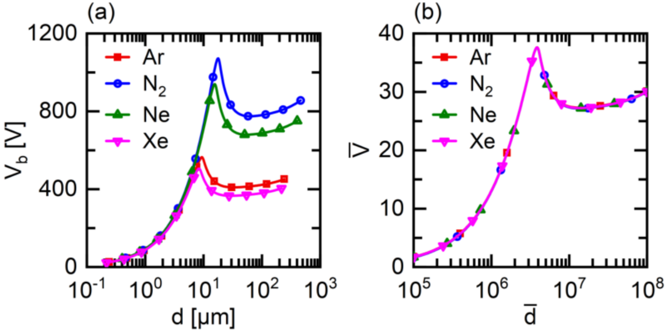
Dimensional analysis

- **Universal Paschen's curve**



$$V_b = \frac{Bpd}{\ln(Apd) - \ln[\ln(1/\gamma_{SE}+1)]}$$

$$\overline{V} = \frac{\overline{p}\overline{d}}{\ln(\overline{p}\overline{d}) - \ln(\ln(1+1/\gamma_{SE}))}$$



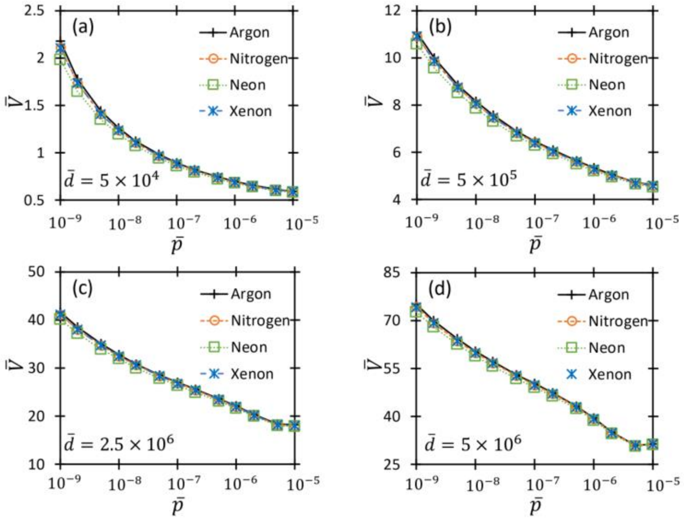


Fig. 18 (a) Comparison of $\bar{V}$ as a function of $\bar{p}$ for argon, nitrogen, neon, and xenon at dimensionless gap distances, $\bar{d}$, of (a) $\bar{d} = 5 \times 10^4$, (b) $\bar{d} = 5 \times 10^5$, (c) $\bar{d} = 2.5 \times 10^6$, and (d) $\bar{d} = 5 \times 10^6$. For a given gap distance, the average percent difference in $\bar{V}$ between each of the gases is 1.6%. As the gap distance increases, the dimensionless breakdown voltage also increases, but the breakdown curves still overlap, which validates the universality of the dimensionless breakdown condition. Reproduced with the permission of AIP Publishing from [42].

A.M. Loveless, A.L. Garner, Phys. Plasmas **24**(11), 113522 (2017)

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Outline

1 • Introduction

2 • Historical development (Breakdown similarity)

3 • Mathematical derivation (F- and B-similarity)

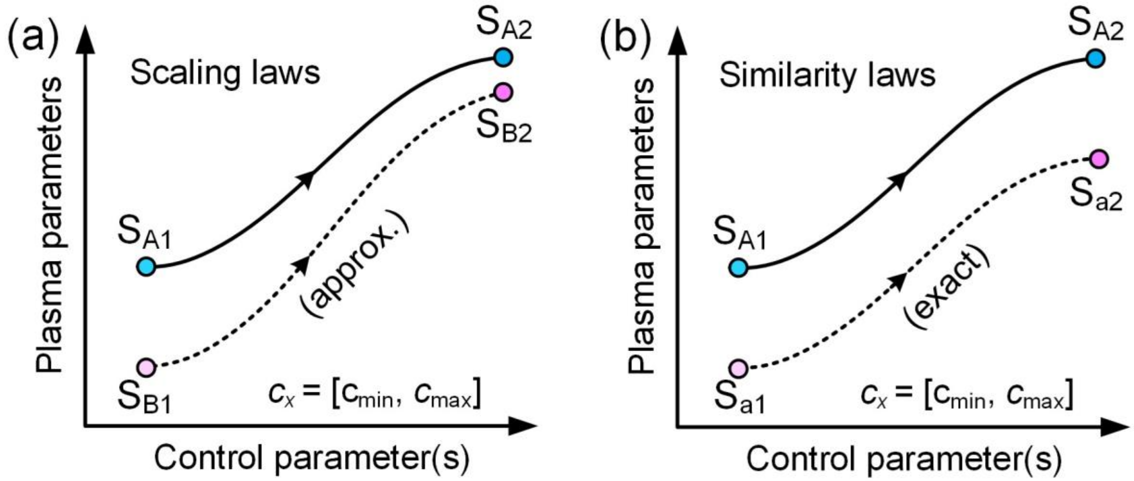
4 • Similarity in discharge plasmas (RF plasmas)

5 • Summary

Similarity versus scaling

- **Discharge parameter and control parameters**

$$G = f(c_1, c_2, c_3, \dots, c_n)$$



$$G = f(c_x)$$

$$G = f(E/p)$$

- Dimensional analysis
- Self-similarity
- Scale-invariance

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Mathematical derivation (1/5)

- **Fluid equations (F-similarity)**

$$\boxed{\begin{gathered}\frac{\partial n_e}{\partial t}+\frac{\partial(n_e\mathbf{v}_e)}{\partial \mathbf{x}}=K_{\mathrm{iz}}n_eN_n,\\ \frac{\partial \mathbf{E}}{\partial \mathbf{x}}=-\frac{q_e}{\epsilon}(n_i-n_e),\end{gathered}}$$

$$n_e(x_1,t_1)=k^{-2}n_e(x_k,t_k)$$

$$G(x_1,t_1)=k^{\alpha[G]}G(x_k,t_k)$$

$$\boxed{\alpha\left[\prod_{j=1}^{m}G_j^{k_j}\right]=\sum_{j=1}^{m}k_j\alpha[G_j]}$$

$$\boxed{\begin{gathered}\frac{\partial(k^{-2}n_e)}{\partial(kt)}+\frac{\partial(k^{-2}n_e\mathbf{v}_e)}{\partial(k\mathbf{x})}=K_{\mathrm{iz}}(k^{-2}n_e)(k^{-1}N_n),\\ \frac{\partial(k^{-1}\mathbf{E})}{\partial(k\mathbf{x})}=-\frac{q_e}{\epsilon}(k^{-2}n_i-k^{-2}n_e).\end{gathered}}$$

$$\boxed{\alpha[K_{\mathrm{iz}}]=0}$$

$$\boxed{K_{\mathrm{iz}}=\sqrt{2/m_e}\int_0^{\infty}\varepsilon\sigma(\varepsilon)f_{\mathrm{EEPF}}(\varepsilon)\mathrm{d}\varepsilon}$$

$$kt,\ k\mathbf{x},\ \mathbf{v}_e,\ k^{-1}N_n,\ k^{-1}\mathbf{E},\ k^{-2}n_e,\ \text{and}\ k^{-2}n_i$$

Y. Fu, et al., IEEE Trans. Plasma Sci. **47**, 1994 (2019)

Mathematical derivation (2/5)

- **Scale-invariant similarity parameters**

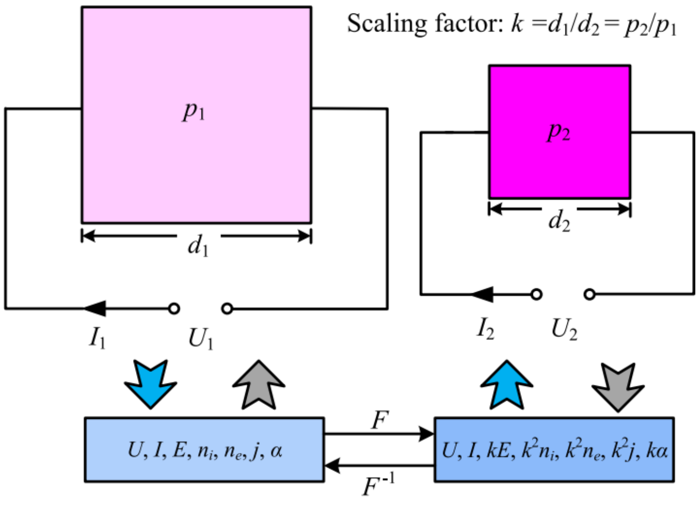
$$G(x_1, t_1) = k^{\alpha[G]} G(x_k, t_k) \qquad \alpha[d] = \alpha[\mathbf{x}] = \alpha[t] = 1$$

$$\alpha[G_1 \cdot G_2] = \alpha[G_1] + \alpha[G_2]$$
$$\alpha[G_1 / G_2] = \alpha[G_1] - \alpha[G_2]$$

$$\alpha[\mathbf{J}] = \alpha[n_e] = \alpha[n_i] = -2$$
$$\alpha[\mathbf{E}] = \alpha[p] = \alpha[\chi] = -1$$

Similarity invariants:

$$\alpha[\mathbf{v}] = \alpha[\mathbf{x}] - \alpha[t] = 0,$$
$$\alpha[\varepsilon] = \alpha[\mathbf{E}] + \alpha[\lambda] = 0$$
$$\alpha[V] = \alpha[\mathbf{E}] + \alpha[d] = 0$$
$$\alpha[\mathbf{E}/p] = \alpha[\mathbf{B}/p] = 0$$
$$\alpha[pd] = \alpha[pt] = \alpha[f/p] = \alpha[fd] = 0$$



Similarity relations: F, F^{-1}

Y. Fu, et al., IEEE Trans. Plasma Sci. **47**, 1994 (2019)

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Mathematical derivation (3/5)

- **Kinetic equations (F-similarity)**

$$\frac{\partial f_e}{\partial t} + \mathbf{v}\cdot\frac{\partial f_e}{\partial \mathbf{x}} + \frac{q_e(\mathbf{E}+\mathbf{v}\times\mathbf{B})}{m_e}\cdot\frac{\partial f_e}{\partial \mathbf{v}} = \sum_j C_{en}^j[f_e f_n, v_{en}, \sigma_{en}(v_{en})] \qquad \frac{\partial \mathbf{E}}{\partial \mathbf{x}} = -\frac{q_e}{\epsilon}\int (f_e - f_i)\mathrm{d}\mathbf{v}$$

$$\frac{\partial(k^{-2}f_e)}{\partial(kt)} + \mathbf{v}\cdot\frac{\partial(k^{-2}f_e)}{\partial(k\mathbf{x})} + \frac{q_e(k^{-1}\mathbf{E}+\mathbf{v}\times k^{-1}\mathbf{B})}{m_e}\cdot\frac{\partial(k^{-2}f_e)}{\partial \mathbf{v}} = \sum_j C_{en}^j[(k^{-2}f_e)(k^{-1}f_n), v_{en}, \sigma_{en}(v_{en})]\,. \qquad \frac{\partial(k^{-1}\mathbf{E})}{\partial(k\mathbf{x})} = -\frac{q_e}{\epsilon}\int (k^{-2}f_e - k^{-2}f_i)\,\mathrm{d}\mathbf{v}$$

⇨ $\boxed{kt,\;\; k\mathbf{x},\;\; k^{-1}\mathbf{E},\;\; k^{-1}\mathbf{B},\;\; k^{-2}f_e}$

$$\alpha[f_e] = \alpha[f_i] = -2 \;\Rightarrow\; \alpha[n_e] = -2$$

$$\Rightarrow\; \alpha[f_\mathrm{p}] = \alpha[f_e/n_e] = 0$$

$$N_n = \int f_n(\mathbf{x},\mathbf{v},t)\,\mathrm{d}^3\mathbf{v} \propto p$$

$$\Rightarrow\; \alpha[f_n] = \alpha[N_n] = \alpha[p] = -1$$

Mathematical derivation (4/5)

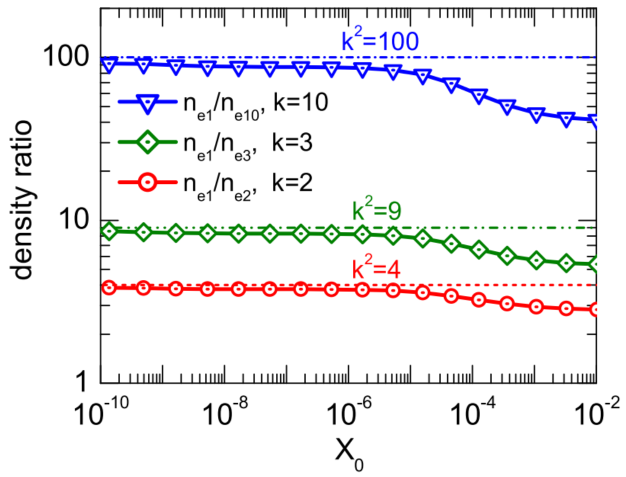
- **Violation of F-similarity: (approaching fully ionization degree)**

$$\alpha[n_e] = -2 \text{ and } \alpha[N_n] = \alpha[p] = -1$$

$$\Rightarrow \quad \alpha[\chi] = \alpha[n_e/N_n] = -1$$

$$\boxed{1 \leftarrow \chi_1 = \left(\frac{n_e}{N_n}\right)_1 = \frac{1}{k}\left(\frac{n_e}{N_n}\right)_k = \frac{1}{k}\chi_k \to 1}$$

$$\text{does not hold if } k \neq 1$$



Deviation from F-similarity as ionization degree increases

Y. Fu, et al., Plasma Sources Sci. Technol. **28**, 095012 (2019)

清華大学 Tsinghua University

Mathematical derivation (5/5)

- **Kinetics equations (B-similarity: high-density/strongly ionized)**

$$\frac{\partial f_e}{\partial t}+\mathbf{v}\cdot\frac{\partial f_e}{\partial \mathbf{x}}+\frac{q_e(\mathbf{E}+\mathbf{v}\times\mathbf{B})}{m_e}\cdot\frac{\partial f_e}{\partial \mathbf{v}}=\sum_j C_{ei}^j[f_e f_i, v_{ei}, \sigma_{ei}(v_{ei})]$$

$$\mathbf{E}\approx-\frac{D_e}{\mu_e}\frac{1}{n_e}\frac{\partial n_e}{\partial \mathbf{x}}$$

$$\frac{\partial(k^{-1}f_e)}{\partial(kt)}+\mathbf{v}\cdot\frac{\partial(k^{-1}f_e)}{\partial(k\mathbf{x})}+\frac{q_e(k^{-1}\mathbf{E}+\mathbf{v}\times k^{-1}\mathbf{B})}{m_e}\cdot\frac{\partial(k^{-1}f_e)}{\partial \mathbf{v}}=\sum_j C_{ei}^j[(k^{-1}f_e)(k^{-1}f_i), v_{ei}, \sigma_{ei}(v_{ei})],$$

$$k^{-1}\mathbf{E}\approx-T_e\frac{1}{(k^{-1}n_e)}\frac{\partial(k^{-1}n_e)}{\partial(k\mathbf{x})}$$

$$\boxed{k^{-1}\mathbf{E}\approx-T_e\frac{1}{(k^{-2}n_e)}\frac{\partial(k^{-2}n_e)}{\partial(k\mathbf{x})}}$$

⇨ $kt,\ k\mathbf{x},\ k^{-1}\mathbf{E},$ and $k^{-1}\mathbf{B}$

$$\boxed{\alpha\left[\frac{\partial \mathbf{E}}{\partial \mathbf{x}}\right]=-2\neq\alpha[\rho]=-1}$$

$$\alpha[n_e]=\alpha[f_e]=\alpha[n_i]=\alpha[f_i]=-1$$

$\alpha[\chi]=\alpha[n_e/N_n]=0$ is satisfied since $\alpha[N_n]=-1$

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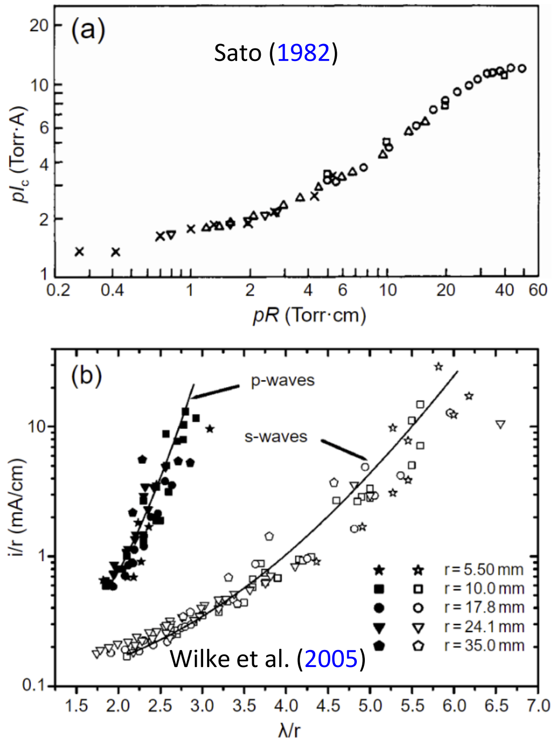
Similarity factors

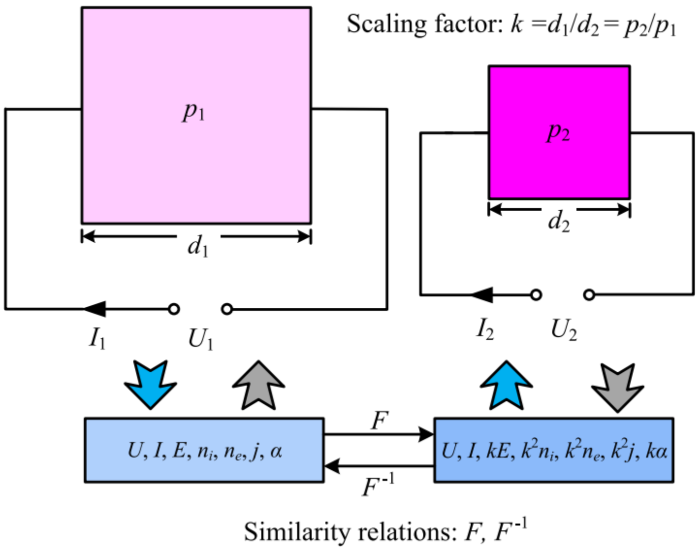
● Comparison of F-similarity and B-similarity

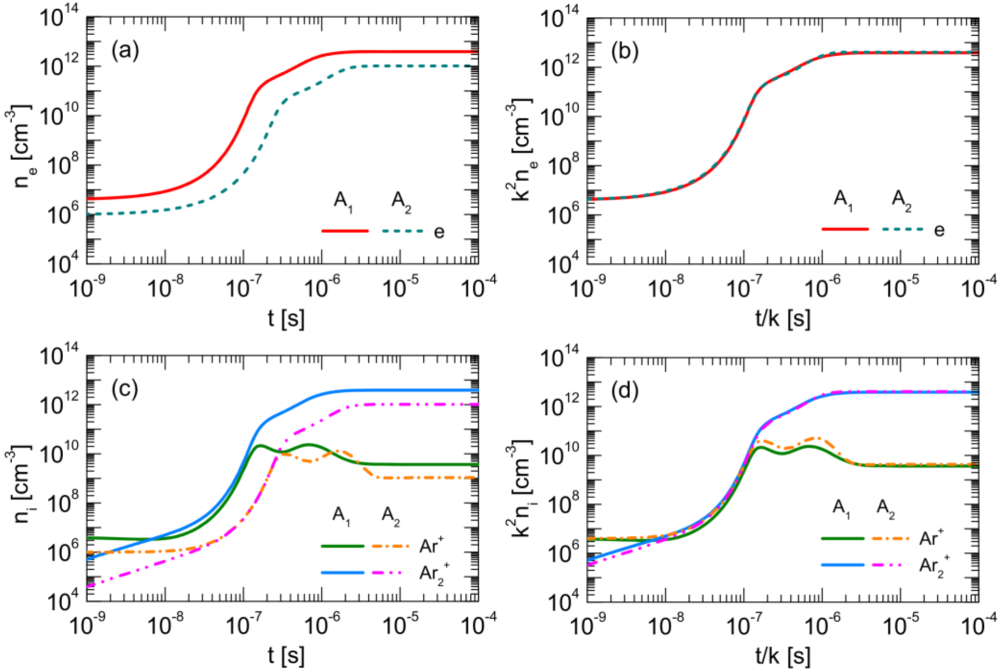
Physical Parameter	F-similarity factor	B-similarity factor
Time, t [s]	$\alpha[t] = 1$	$\alpha[t] = 1$
Position, $\mathbf{x}$ [m]	$\alpha[\mathbf{x}] = 1$	$\alpha[\mathbf{x}] = 1$
Dimension, d [m]	$\alpha[d] = 1$	$\alpha[d] = 1$
Mean free path, λ [m]	$\alpha[\lambda] = 1$	$\alpha[\lambda] = 1$
Current, I [A]	$\alpha[I] = 0$	$\alpha[I] = 1$
Voltage, V [V]	$\alpha[V] = 0$	$\alpha[V] = 0$
Velocity, $\mathbf{v}$ [m/s]	$\alpha[\mathbf{v}] = 0$	$\alpha[\mathbf{v}] = 0$
Energy, ε [eV]	$\alpha[\varepsilon] = 0$	$\alpha[\varepsilon] = 0$
EEPF, f_{p} [$\mathrm{eV}^{-3/2}$]	$\alpha[f_{\mathrm{p}}] = 0$	$\alpha[f_{\mathrm{p}}] = 0$
Pressure, p [Pa]	$\alpha[p] = -1$	$\alpha[p] = -1$
Frequency, f [Hz]	$\alpha[f] = -1$	$\alpha[f] = -1$
Electric field, $\mathbf{E}$ [V/m]	$\alpha[\mathbf{E}] = -1$	$\alpha[\mathbf{E}] = -1$
Magnetic field, $\mathbf{B}$ [T]	$\alpha[\mathbf{B}] = -1$	$\alpha[\mathbf{B}] = -1$
Electron density, n_e [$1/\mathrm{m}^3$]	$\alpha[n_e] = -2$	$\alpha[n_e] = -1$
Ion density, n_i [$1/\mathrm{m}^3$]	$\alpha[n_i] = -2$	$\alpha[n_i] = -1$
Space charge, ρ [$\mathrm{C/m}^3$]	$\alpha[\rho] = -2$	$\alpha[\rho] = -1$
Current density, $\mathbf{J}$ [$\mathrm{A/m}^2$]	$\alpha[\mathbf{J}] = -2$	$\alpha[\mathbf{J}] = -1$



Dynamical similarity (1/2)

- **Transient processes:**





Y. Fu, et al. IEEE Trans. Plasma Sci. **47**, 1994 (2019)

Y. Fu, et al. Plasma Sources Sci. Technol. **28**, 095012 (2019)

Dynamical similarity (2/2)

- **Steady-state glow discharges:**

Fig. 20 Voltage-current characteristics of discharges with different pressure-distance products pd **a** $pd = 150$ Pa · cm; **b** $pd = 75$ Pa · cm; and **c** $pd = 45$ Pa · cm. The voltage data are equal to the gap voltage minus the breakdown voltage U_b. Reproduced with the permission of IOP Publishing from Marić et al. (2003)

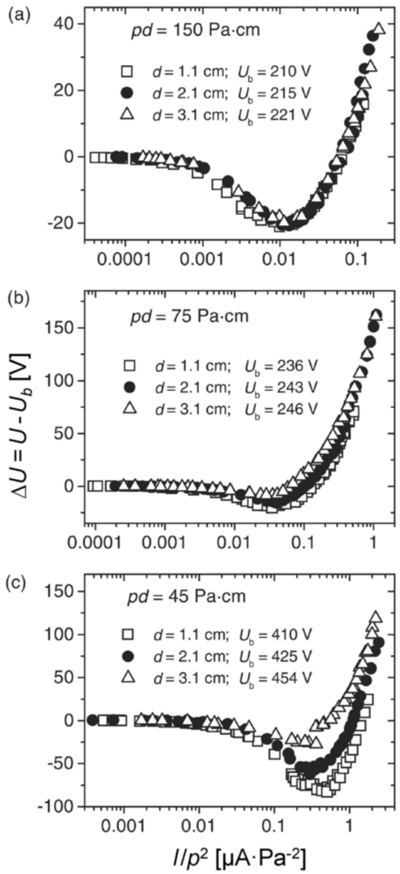
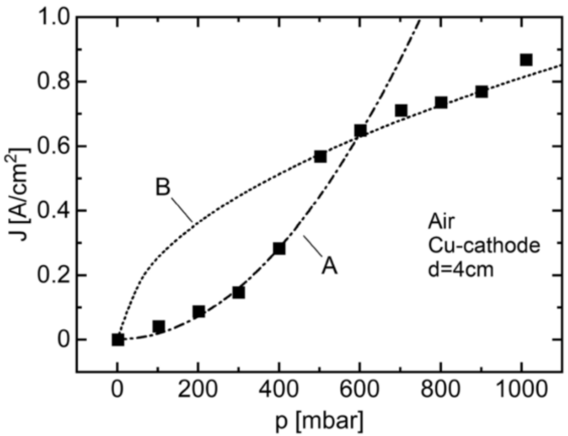


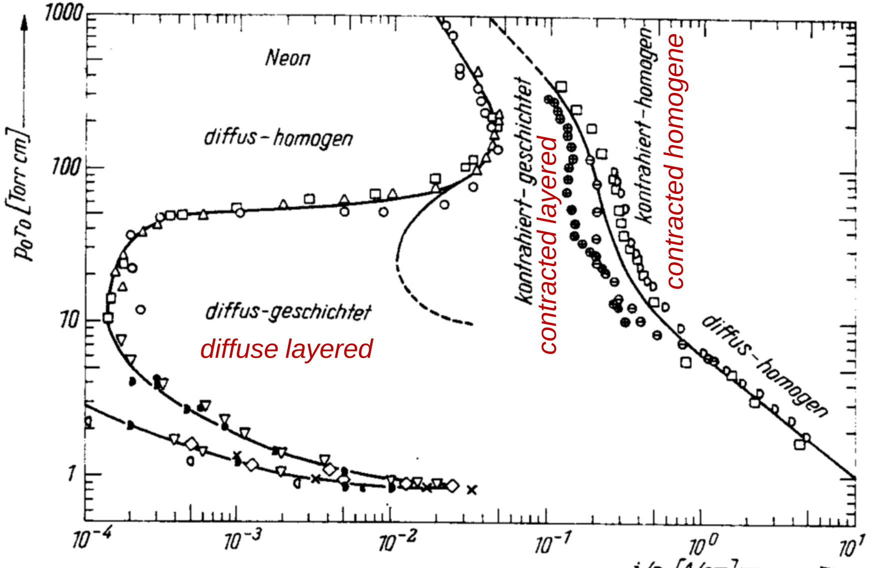
Fig. 21 Current density scaling with the gas pressure p [unit in mbar, 1 mbar = 100 Pa], with the filled squares experimentally measured in air. Curve A is a quadratic fitting $J \propto p^2$, and curve B is a square-root fitting $J \propto \sqrt{p}$. Reproduced with the permission of IOP Publishing from Mezei et al. (1998)



[1] Y. Fu*, et al., Reviews of Modern Plasma Physics **7**, pp.1-81 (2023).

Similarity map

- **B-similarity: *pR* and *i/R* parameter map**

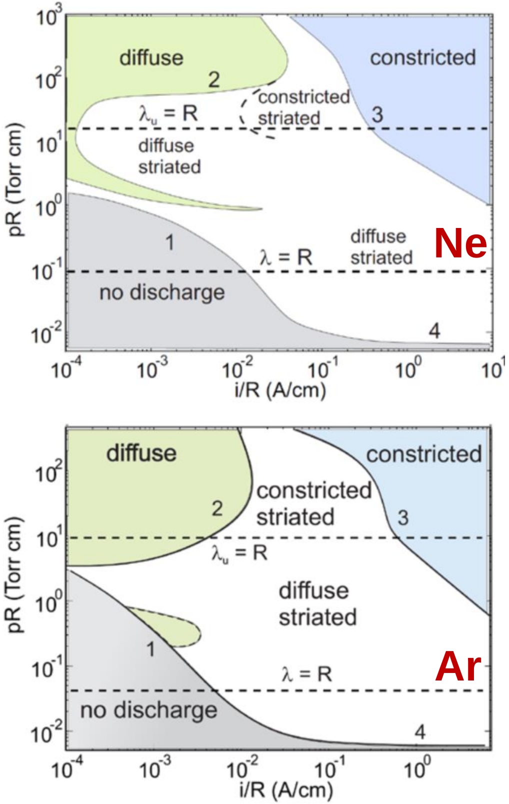


S. Pfau, et al., Beitr. Plasmaphys. **9**, 333 (1969).

Abb. 8. Existenzgrenzen laufender Schichten in Neon

× : $r_0 = 0{,}3$ cm; ◇ : $r_0 = 0{,}4$ cm; ▽ : $r_0 = 0{,}5$ cm; △ : $r_0 = 0{,}75$ cm; ◗ : $r_0 = 1{,}0$ cm;

□ : $r_0 = 1{,}5$ cm; ◖ : $r_0 = 2{,}0$ cm; ∘ : $r_0 = 3{,}0$ cm; ⊖ : $r_0 = 3{,}7$ cm; ⊕ : $r_0 = 7{,}0$ cm; [32].



V. I. Kolobov, et al., Phys. Rev. E **106**, 065206 (2022).

Application of similarity laws

- **F- and B-similarity in discharges:**

F-similarity	B-similarity
Paschen (1889), Townsend (1915) [DC breakdown]	White (1959) [HCD], Sturges and Oskam (1964) [He-Ne]
Holm (1924) [Current], Engel et al. (1933) [Steady-state glow]	Gordon and White (1963) [He-Ne gas laser]
Margenau and Hartman (1948), Llewellyn Jones and Morgan (1951) [RF, MW breakdown]	Muehe 1974, Dote 1976, Dote and Ichikawa (1976) [Glow PC]
Brown (1959) [2f-RF], Muehe (1965) [AC map], Woo and Ishimaru (1967) [Multipactor]	Lotkova and Sokovikov (1983) [CO laser] Lotkova and Ponomarev (1988) [CO_2 laser]
Bortnik and Cooke (1972) [SF6-EHV], Phelps (1990) [AC, DC]	Bashlov and Timofeev (1991) [Pulse discharge, gas + metal vapor]
Surendra and Graves (1991), Mezei et al. (1998) [Glow-liquid]	Rukhadze et al. (1991), Sokovikov (2000) [HF molecular gas laser]
Liu and Pasko (2006) [Streamer], Mesyats 2006b [PS breakdown] Dekić et al. (2010) [Pulse breakdown]	Wilke et al. (2005) [S/P-Waves, Glow PC]
Matthias et al. (2020) [Thruster], Lee et al. (2017) [Microgap RF], Fu et al (2019) [RF]; Ryutov (2018) [MHD], Yao et al. 2022 [EMP]	D. Michael et al (2010) [Light source diagnostics]; Kolobov and Arslanbekov 2022 [Striation/ ionization waves]

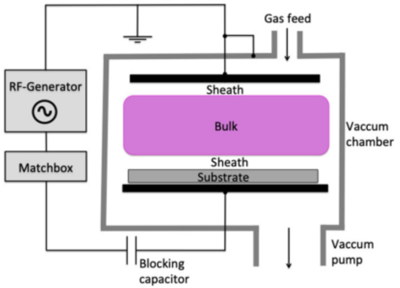
Outline

1. Introduction
2. Historical development (Breakdown similarity)
3. Mathematical derivation (F- and B-similarity)
4. Similarity in discharge plasmas (RF plasmas)
5. Summary

RF plasmas

• Rf plasma sources and application

$$G = f(p, d, f, V_{\mathrm{ap}}, P_{\mathrm{abs}}, \ldots)$$

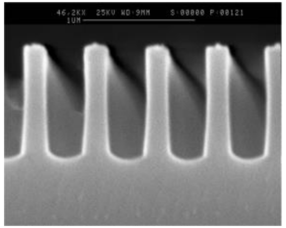


Rf plasma

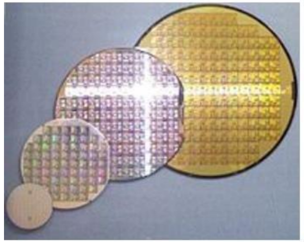
Etching device

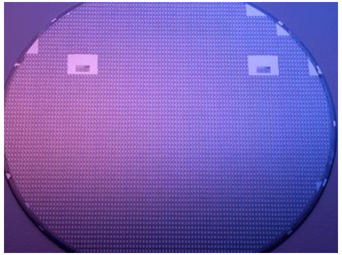
Plasma etching

Plasma etching



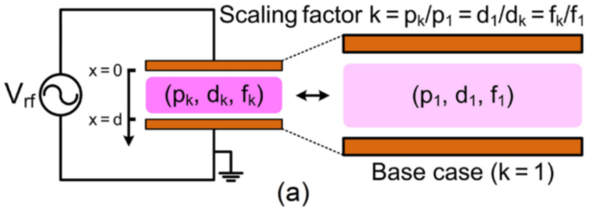
Microstructure





Increasing wafer size

Dimensional Scaling



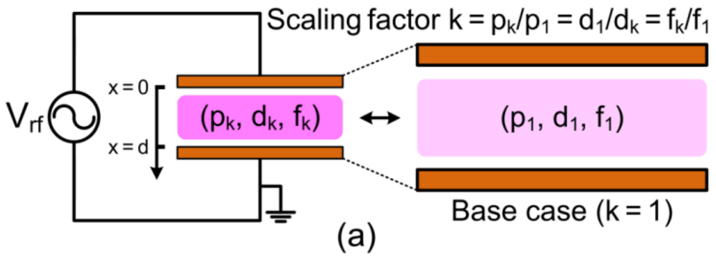
[1] S. Wilczek, et al. J. Appl. Phys. **127**, 181101 (2020).
[2] Y. Fu, et al. Phys. Plasmas **27**, 113501 (2020).

Nonlocal kinetic regimes

- **PIC simulation** (1d3v, electrostatic), **argon**
- Base case (k = 1):

  $[p_1, d_1, f_1]$ = [5 mTorr, 10 cm, 13.56 MHz]

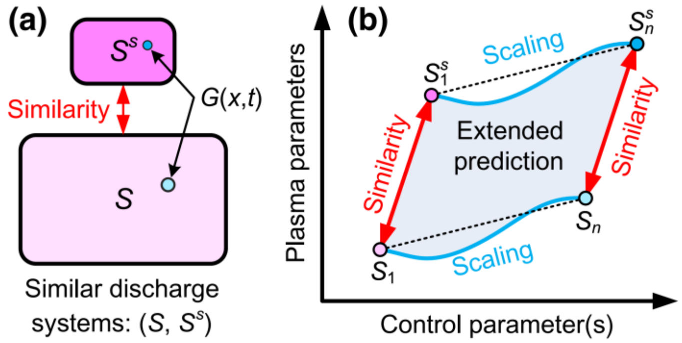
  Driving voltage: $V_{rf}(t) = 300 \cdot \sin(2\pi ft)$



- Electron energy relaxation length (> 200 cm)

$$\lambda_\varepsilon = \lambda_{el}\left[\frac{2m_e}{M} + \frac{2}{3}\left(\frac{\varepsilon_{exc}}{k_B T_e}\right)\frac{\nu_{exc}}{\nu_m} + \frac{2}{3}\left(\frac{\varepsilon_{iz}}{k_B T_e}\right)\frac{\nu_{iz}}{\nu_m} + 3\frac{\nu_{iz}}{\nu_m}\right]^{-1/2}$$

- Highly nonlocal: $\lambda_\varepsilon \sim 200\ \mathrm{cm} >> d$



Y. Fu, et al., Phys. Plasmas **27**, 115501 (2020).

Y. Fu, et al., Phys. Rev. Applied **16**, 054016 (2021).

Electron kinetics

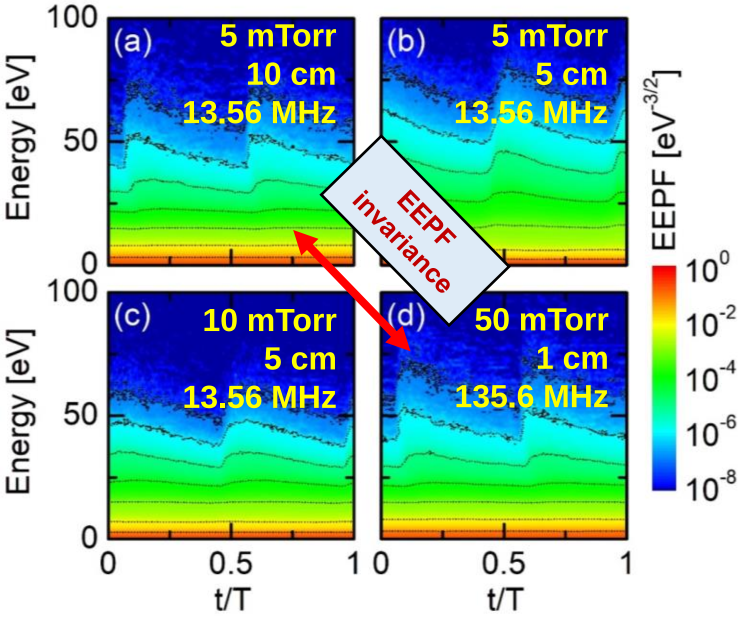
- **Plasma collisionalities:**

Collision in space domain

$$pd \propto d/\lambda \sim N_{coll}$$

Collision in time domain

$$f/p \propto f/\nu_{coll} = T^{-1}/\tau_{coll}^{-1} \sim N_{coll}^{-1}$$



PHYSICAL REVIEW VOLUME 73, NUMBER 4 FEBRUARY 15, 1948

Theory of High Frequency Gas Discharges. IV. Note on the Similarity Principle[1]

H. MARGENAU
Yale University, New Haven, Connecticut

(Received August 18, 1947)

The similarity principle of Steenbeck is derived from the differential equation for the electron distribution function and is extended to cover h.f. discharges. Possible applications and its limitations are discussed.

The very early similarity theory for HF discharges!

currents is not very useful. Hence we shall here define similar discharges to mean discharges in which ***the distribution in energy of the electrons at corresponding points of space is the same***. Whether

With local field assumption!

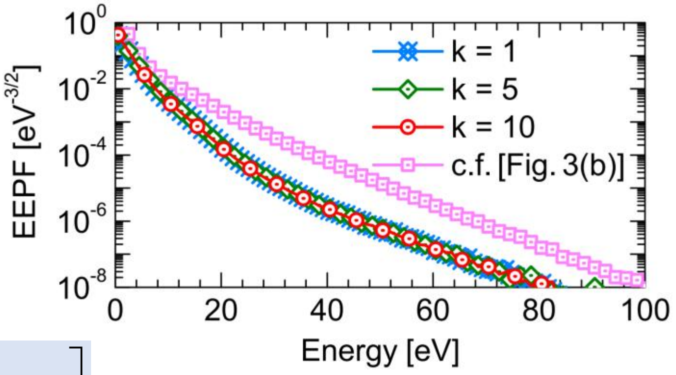
Tsinghua University

Electron Boltzmann equation

- **Boltzmann equation (weakly ionized):**

$$\frac{\partial f_e}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}} f_e - \frac{e\mathbf{E}}{m_e}\cdot\nabla_{\mathbf{v}} f_e = \sum_j C_{ej}\left[f_e f_j, v_{ej}, \sigma_{ej}(v_{ej}) \right]$$

$$\frac{\partial(f_e/p^2)}{\partial(pt)} + \mathbf{v}\cdot\nabla_{(p\mathbf{x})}(f_e/p^2) - \frac{e(\mathbf{E}/p)}{m_e}\cdot\nabla_{\mathbf{v}}(f_e/p^2) = C_{en}\left[\frac{f_e f_n}{p^3}, v_{en}, \sigma_{en}(v_{en})\right]$$



$$\boxed{\sqrt{\varepsilon}\, g_{\mathrm{EEPF}}(\varepsilon)\mathrm{d}\varepsilon = 4\pi\mathbf{v}^2 f_e(\mathbf{v})/n_e\, \mathrm{d}\mathbf{v}}$$

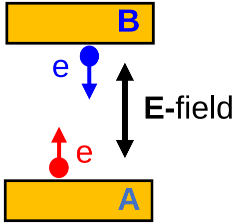
EEPF invariance: $\alpha[g_{\mathrm{EEPF}}(\varepsilon)]=0$

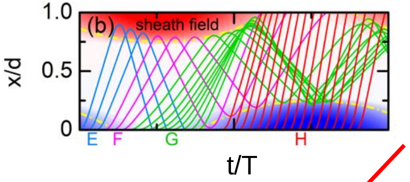
- More exact than the fluidic interpretation!

Y. Fu, et al., Phys. Plasmas **27**, 115501 (2020).

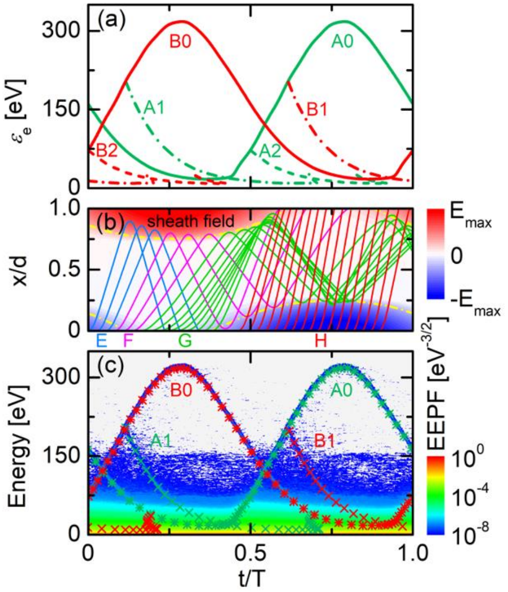
Ballistic electrons (1/2)

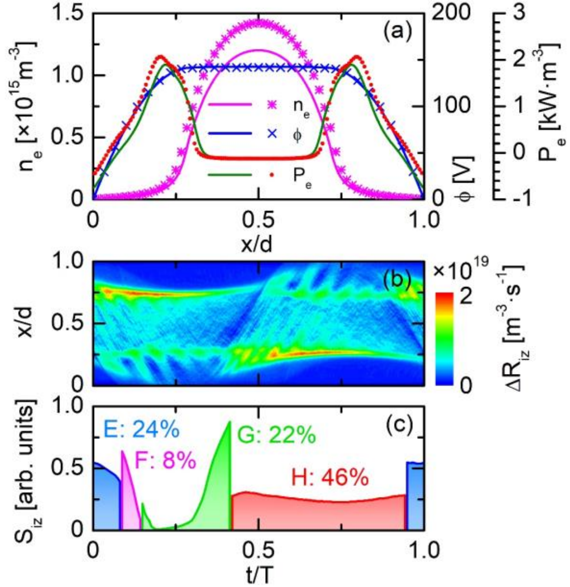
- **Test Particle Simulation: Electrode: A (x = 0), B (x = d)**



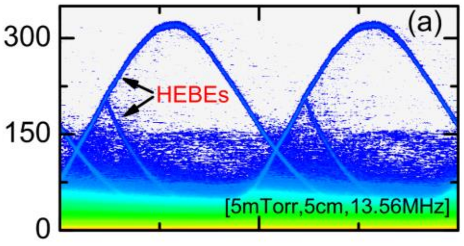


Test particle





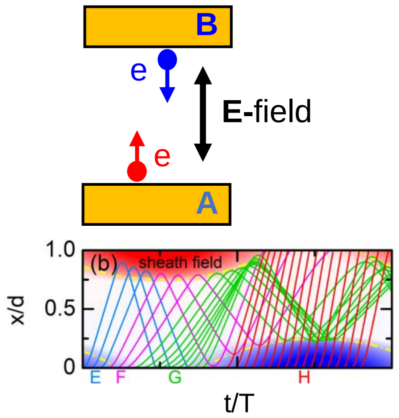
PIC results

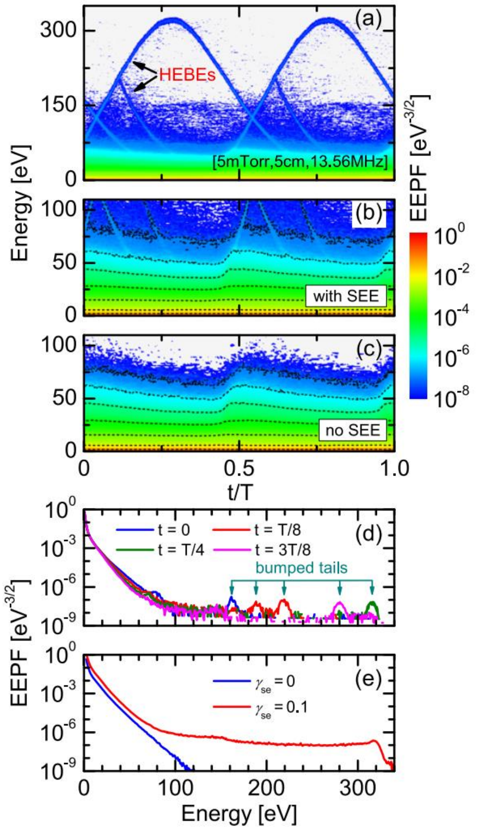


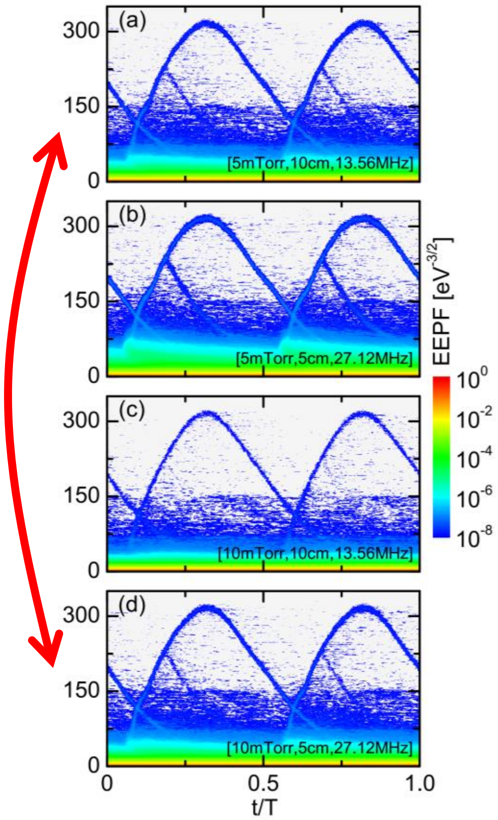
EEPF at the gap center
(Base case)

Y. Fu, et al., Plasma Sources Sci. Technol. **29**, 09LT01 (2020).

Ballistic electrons (2/2)







Beam electron behaviors also follow the similarity relation.

Y. Fu, et al., Plasma Sources Sci. Technol. **29**, 09LT01 (2020).

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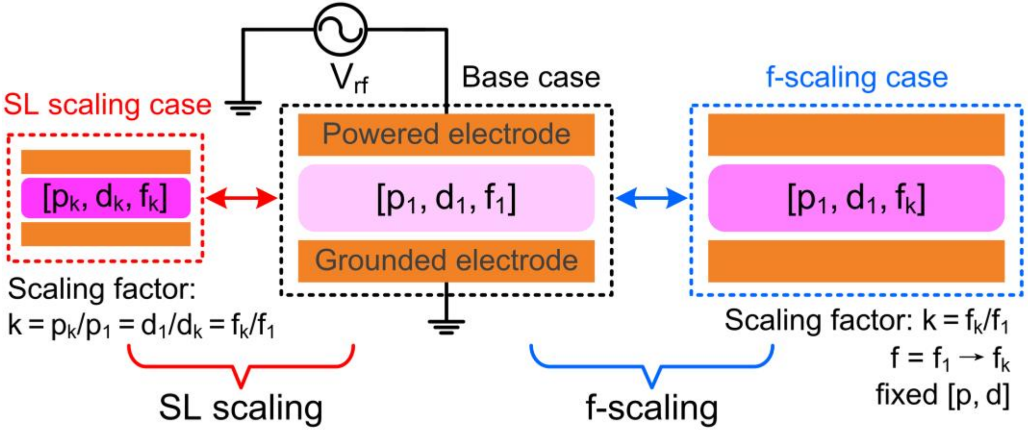
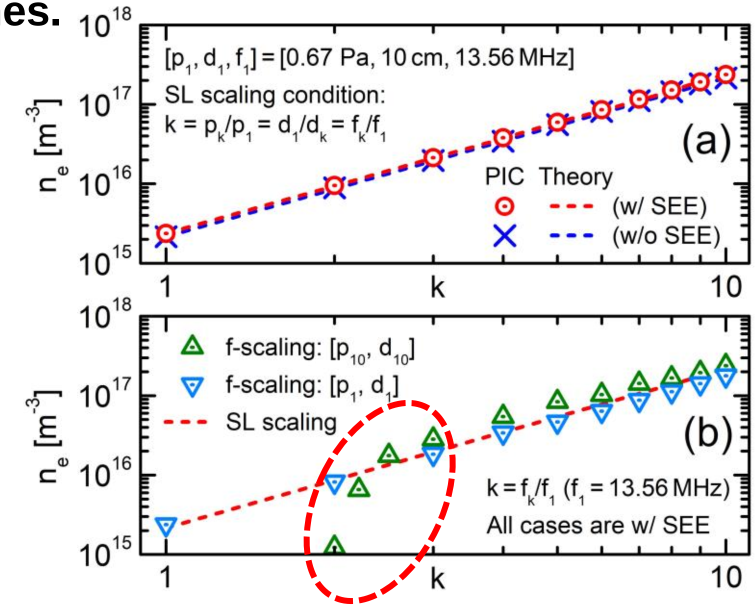
Similarity and scaling (1/4)

- **Similarity law is rigorous while the frequency scaling is with approximations;**
- **The relations are closely the same in certain regimes.**



Multiple control parameters tuned

Single parameter tuned



Similarity law vs frequency scaling

[1] Y. Fu, B. Zheng, D.-Q. Wen, P. Zhang, Q. H. Fan, and J. P. Verboncoeur, Appl. Phys. Lett. **117**, 204101 (2020).

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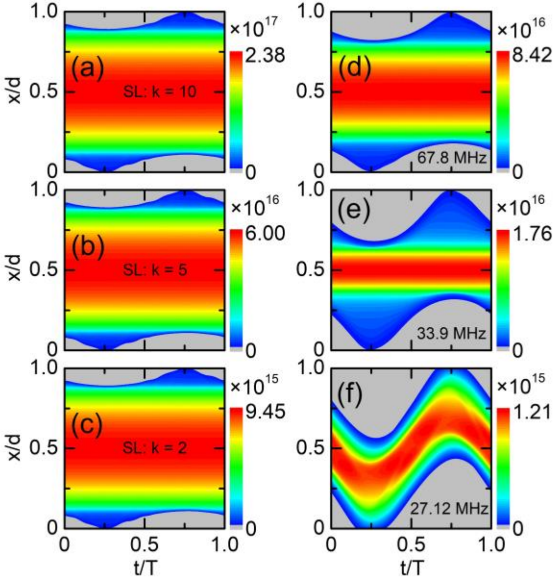
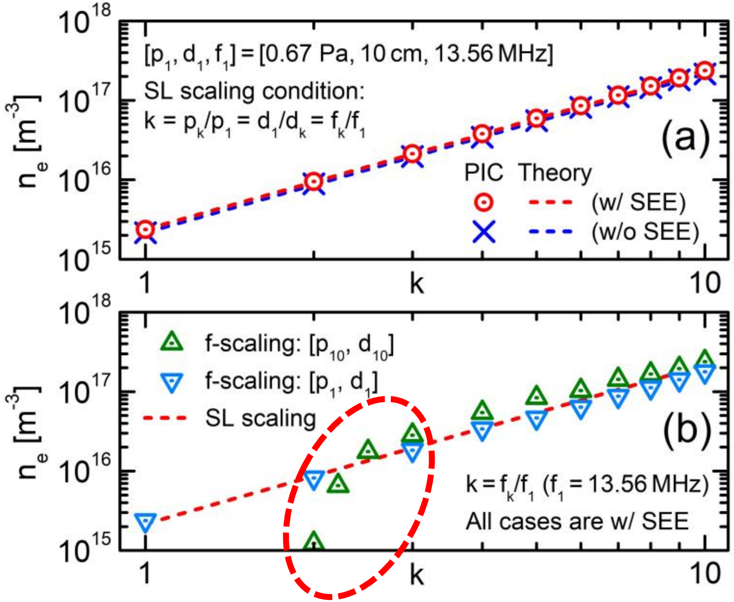
Similarity and scaling (2/4)

• Validation and violation of the frequency scaling



Spatiotemporal electron density

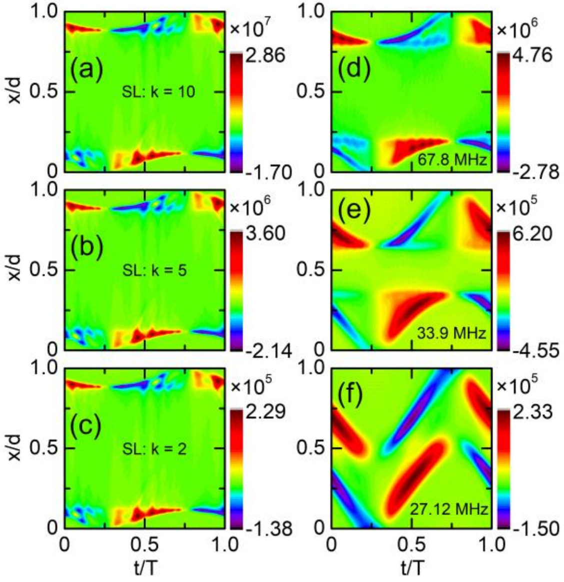


Similarity law vs frequency scaling

[1] Y. Fu, B. Zheng, D.-Q. Wen, P. Zhang, Q. H. Fan, and J. P. Verboncoeur, Appl. Phys. Lett. **117**, 204101 (2020).

Similarity and scaling (3/4)

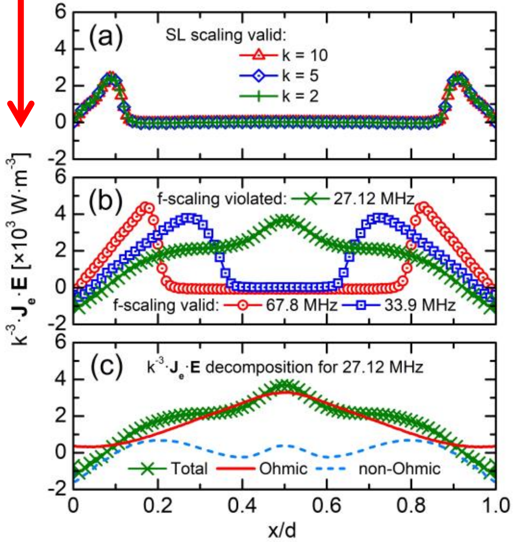
- **Electron heating characteristics**



Electron power absorption

$$P_{\text{abs}} = 2\int_0^{d/2} \mathbf{J}_e \cdot \mathbf{E} dx \approx P_{e\max} \cdot s \propto k^2$$ This work

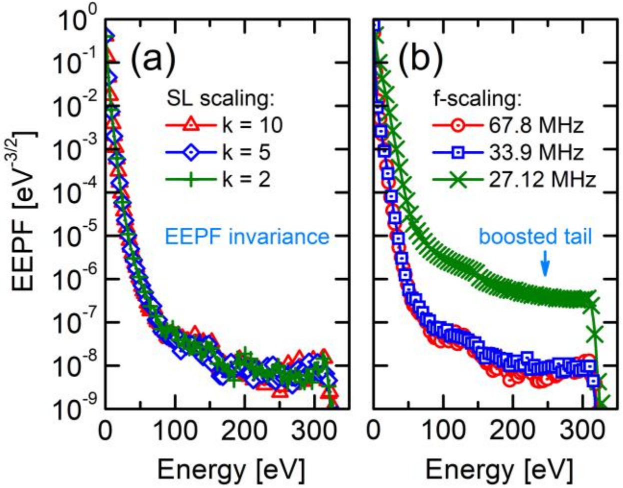
$$P_{\text{abs}} = \widetilde{P}_{\text{abs}}/d = K_{iz}\varepsilon_c(T_e)n_e N_g \propto k^2$$ Lieberman et al.



[1] Y. Fu, B. Zheng, D.-Q. Wen, P. Zhang, Q. H. Fan, and J. P. Verboncoeur, Appl. Phys. Lett. **117**, 204101 (2020).
[2] M. Vass, S. Wilczek, T. Lafleur, R. P. Brinkmann, Z. Donko, and J. Schulze, Plasma Sources Sci. Technol. **29**, 085014 (2020).
[3] V. Vahedi, C. K. Birdsall, M. A. Lieberman, G. DiPeso, and T. D. Rognlien, Phys. Fluids B **5**, 2719 (1993).
[4] M. Surendra and D. B. Graves, Appl. Phys. Lett. **59**, 2091 (1991).
[5] J. K. Lee, O. V. Manuilenko, N. Y. Babaeva, H. C. Kim, and J. W. Shon, Plasma Sources Sci. Technol. **14**, 89 (2005).

Similarity and scaling (4/4)

- **Electron kinetic invariance is the key!**



Electron energy probability functions

[1] Y. Fu, B. Zheng, D.-Q. Wen, P. Zhang, Q. H. Fan, and J. P. Verboncoeur, Appl. Phys. Lett. **117**, 204101 (2020).
[2] M. Vass, S. Wilczek, T. Lafleur, R. P. Brinkmann, Z. Donko, and J. Schulze, Plasma Sources Sci. Technol. **29**, 085014 (2020).
[3] V. Vahedi, C. K. Birdsall, M. A. Lieberman, G. DiPeso, and T. D. Rognlien, Phys. Fluids B **5**, 2719 (1993).
[4] M. Surendra and D. B. Graves, Appl. Phys. Lett. **59**, 2091 (1991).
[5] J. K. Lee, O. V. Manuilenko, N. Y. Babaeva, H. C. Kim, and J. W. Shon, Plasma Sources Sci. Technol. **14**, 89 (2005)

Plasma series resonance (PSR)

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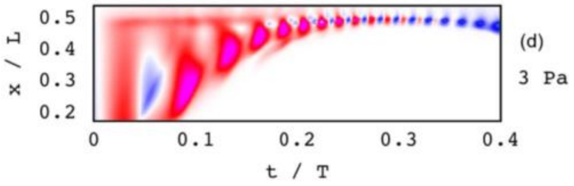
- **Electron heating mechanisms**

$$P_e(x,t) = \mathbf{J}_e(x,t) \cdot \mathbf{E}(x,t)$$



Plasma series resonance (PSR)[1,2]

- **Similarity relation for electron heating**

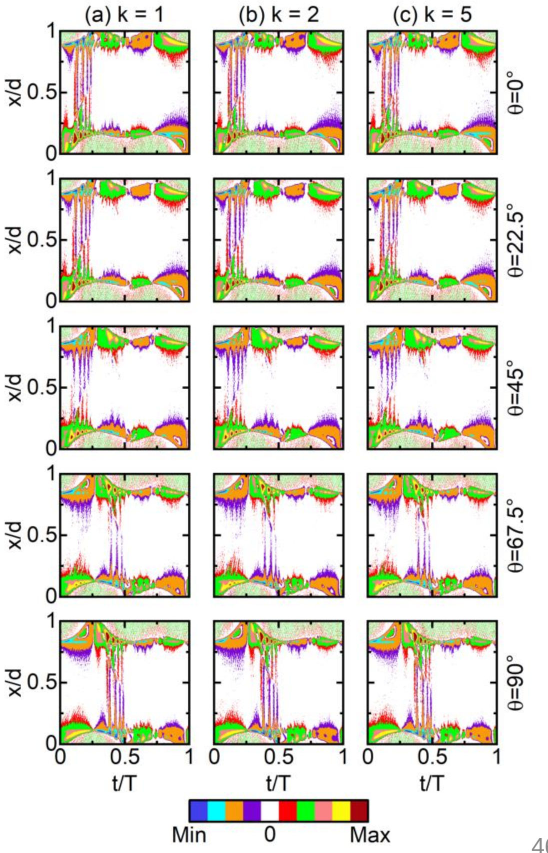
$$\alpha[\mathbf{E}] = \alpha[p] = -1$$

$$\alpha[\mathbf{J}_e] = \alpha[n_e] = -2$$

$$P_e(x,t) = \mathbf{J}_e(x,t) \cdot \mathbf{E}(x,t)$$

$$\alpha[\mathbf{J}_e \cdot \mathbf{E}] = \alpha[\mathbf{J}_e] + \alpha[\mathbf{E}] = -3$$

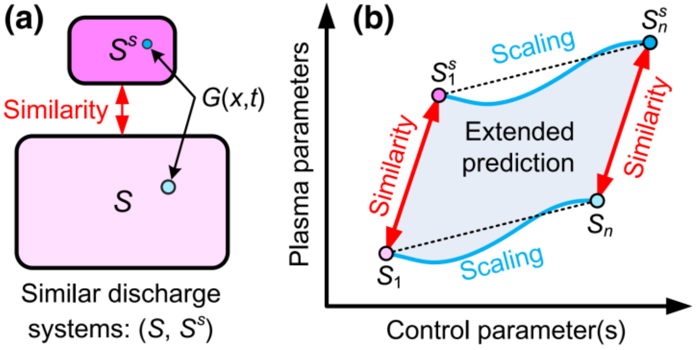
$$P_e(x_1,t_1) = k^{-3} P_e(x_k, x_k)$$

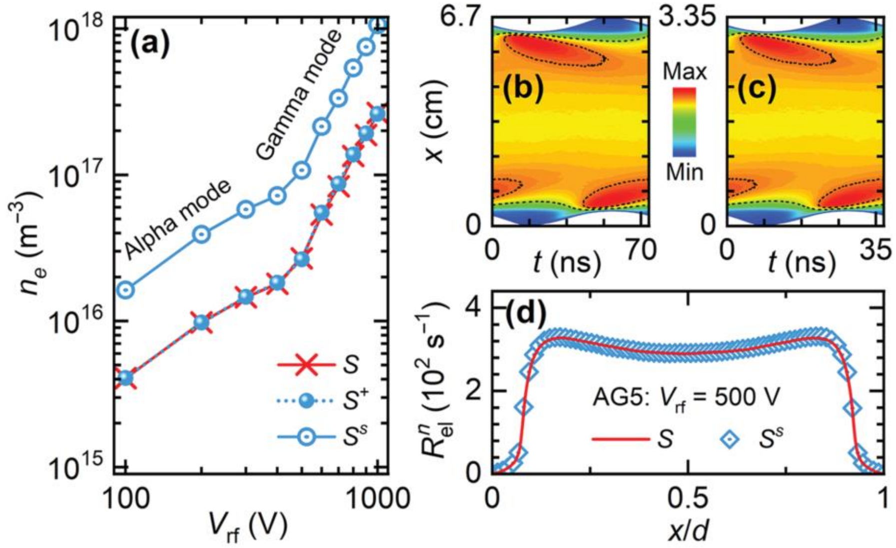


[1] Z. Donkó, J. Schulze, et al., Appl. Phys. Lett. **94**, 131501 (2009).
[2] J. T. Gudmundsson and D. I. Snorrason, J. Appl. Phys. **122**, 193302 (2017).
[3] Y. Fu, et al., Phys. Plasmas **27**, 115501 (2020).

Nonlinear mechanisms (1/3)

- **Alpha-gamma mode transition**

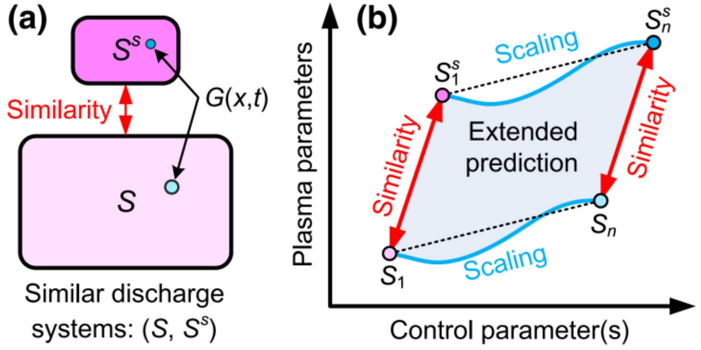


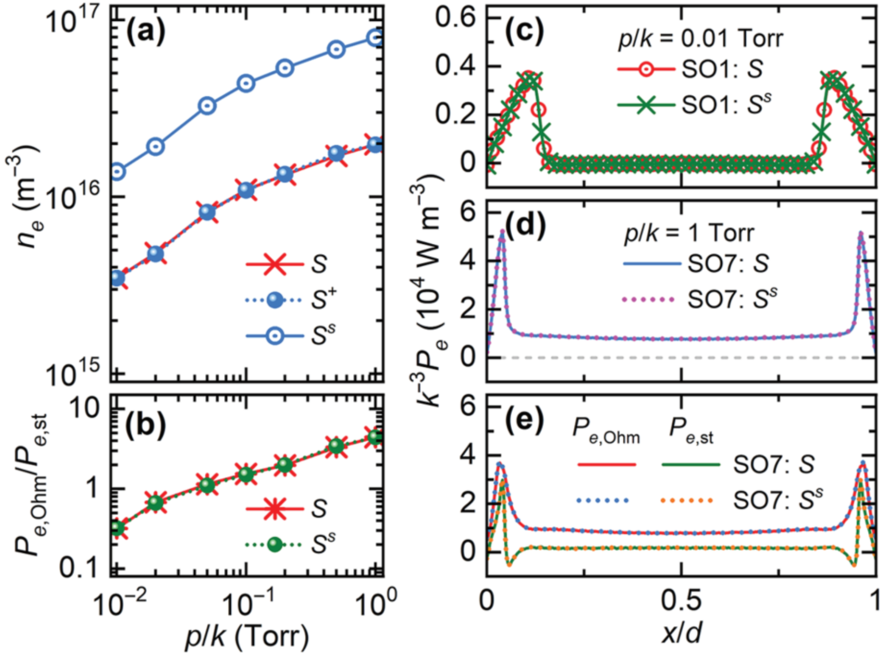


Y. Fu, et al., Phys. Rev. Applied **16**, 054016 (2021).

Nonlinear mechanisms (2/3)

- **Stochastic-Ohmic-heating mode**

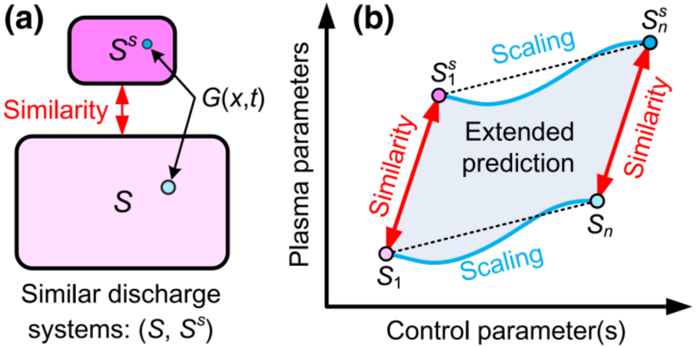


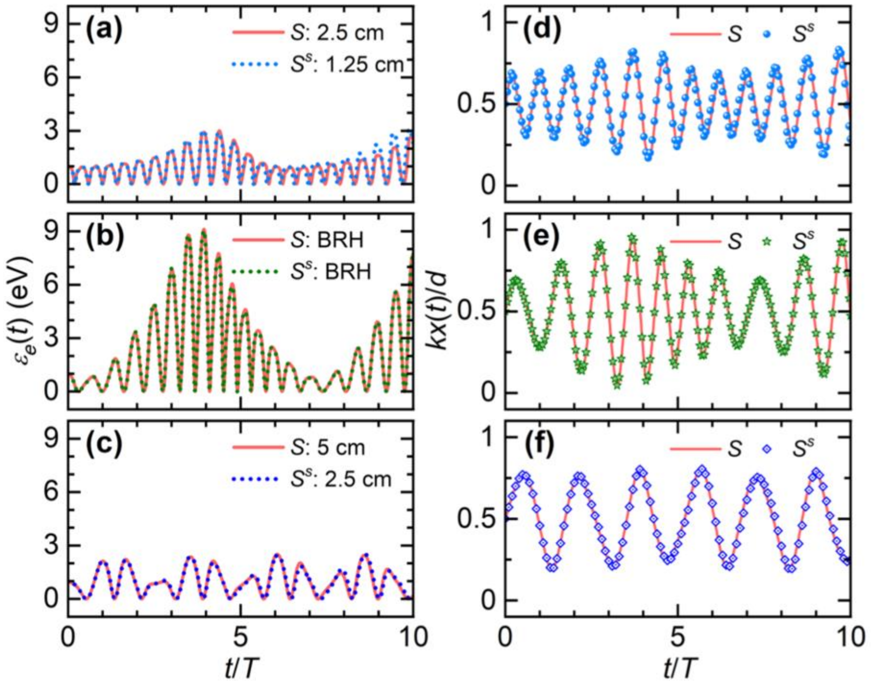


Y. Fu, et al., Phys. Rev. Applied **16**, 054016 (2021).

Nonlinear mechanisms (3/3)

- **Bounce-resonance-heating (BRH)**





Y. Fu, et al., Phys. Rev. Applied **16**, 054016 (2021).

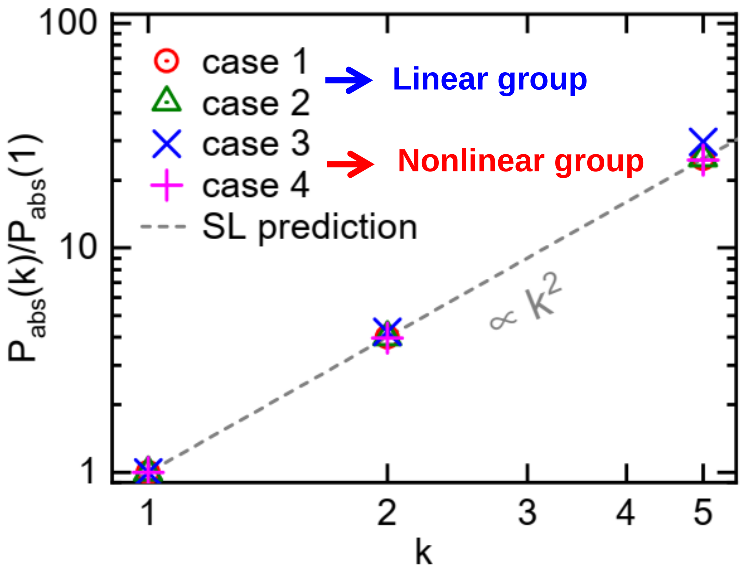
Nonlinear collision (1/2)

- **Effects of collision processes:**
  - **Linear collision: R1-R6**
  - **With nonlinear collision: R1-R15**

Nos.	Collision processes
R1	$e + Ar \rightarrow e + Ar$
R2	$e + Ar \rightarrow 2e + Ar^+$
R3	$e + Ar \rightarrow e + Ar(4s)$
R4	$e + Ar \rightarrow e + Ar(4p)$
R5	$Ar^+ + Ar \rightarrow Ar^+ + Ar$
R6	$Ar^+ + Ar \rightarrow Ar + Ar^+$
R7	$e + Ar(4s) \rightarrow e + Ar$
R8	$e + Ar(4p) \rightarrow e + Ar$
R9	$e + Ar(4s) \rightarrow e + Ar(4p)$
R10	$e + Ar(4p) \rightarrow e + Ar(4s)$
R11	$e + Ar(4s) \rightarrow 2e + Ar^+$
R12	$e + Ar(4p) \rightarrow 2e + Ar^+$
R13	$Ar(4s) + Ar(4s) \rightarrow e + Ar + Ar^+$
R14	$Ar + Ar(4s) \rightarrow Ar + Ar(4s)$
R15	$Ar + Ar(4p) \rightarrow Ar + Ar(4p)$

Stepwise ionizations

D. Yang, et al., Plasma Sources Sci. Technol. **30**, 115009 (2021).

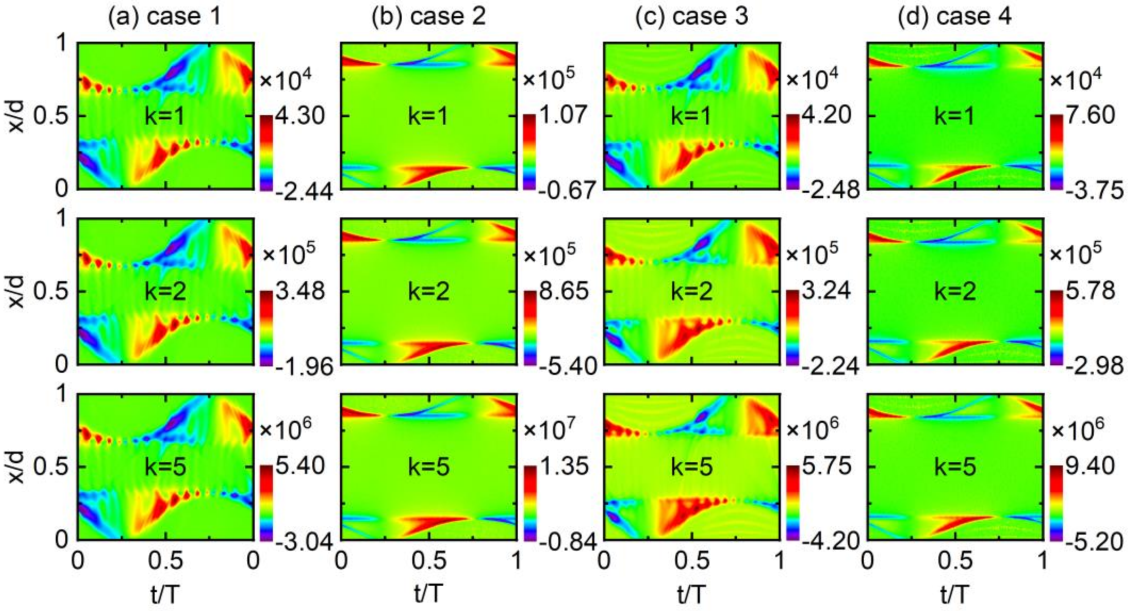


Scaled electron power absorption

p: 5 – 500 mTorr with k = 1, 2 and 5

Nonlinear collision (2/2)

- **Electron heating: w/ and w/o stepwise ionization**



Electron Power Absorption
$P_e = J_eE$

p = 5 – 500 mTorr
k = 1, 2, 5

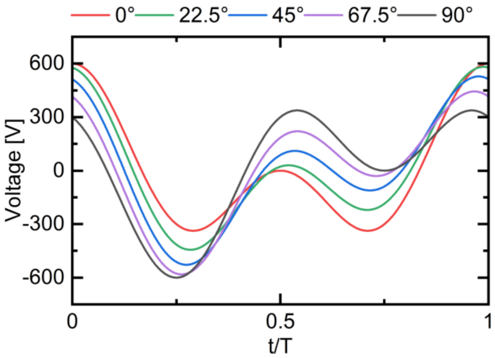
Low pressure → High pressure **Low pressure → High pressure**

Linear collision **Nonlinear collision**

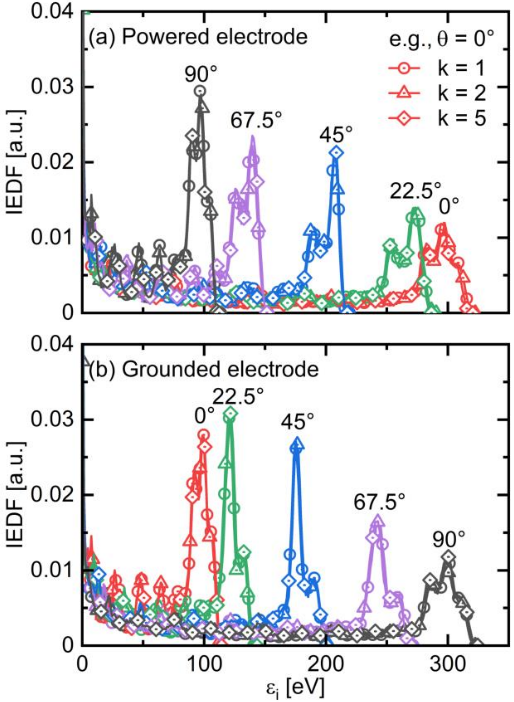
D. Yang, et al., Plasma Sources Sci. Technol. **30**, 115009 (2021).

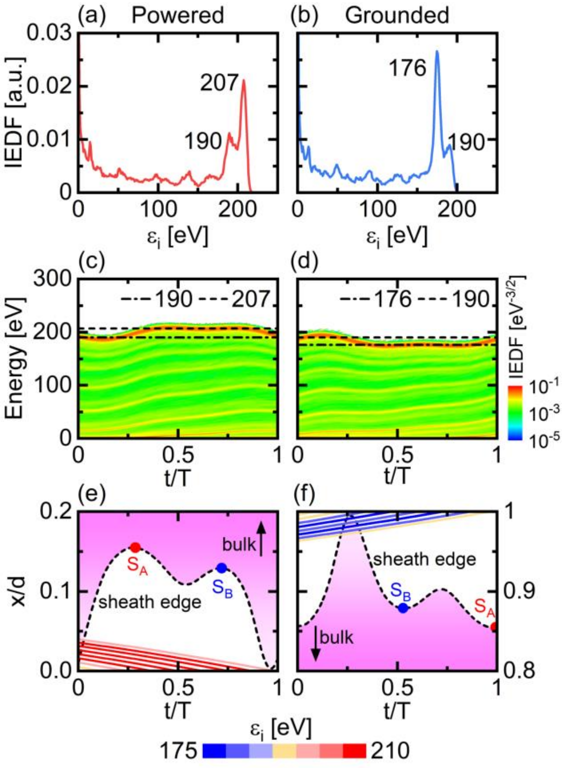
Electrical asymmetry effect

● Dual-frequency RF plasmas



Dual-frequency voltage waveforms



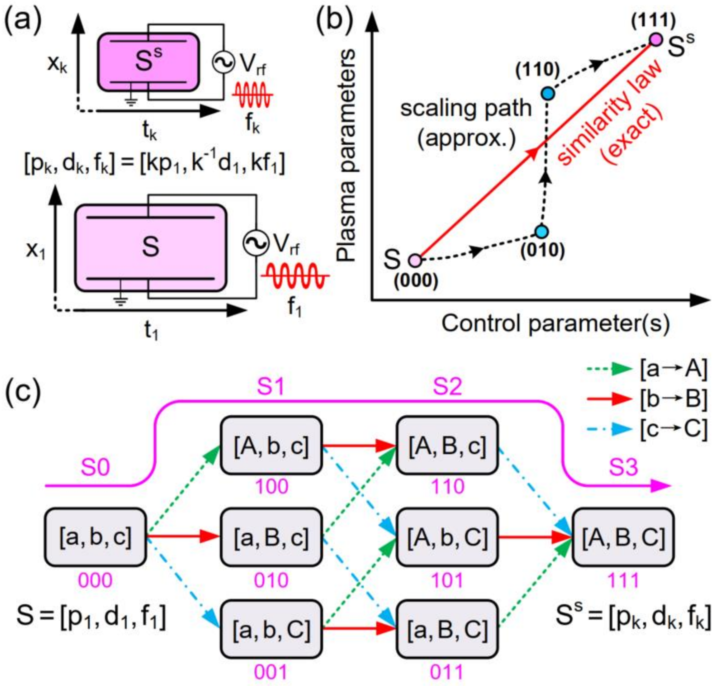


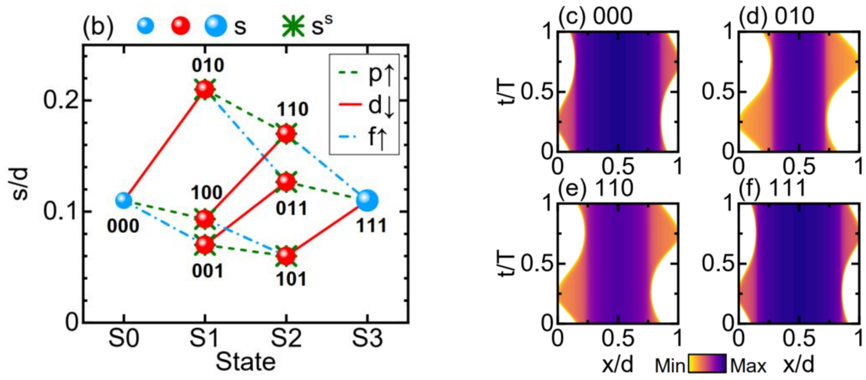
D. Yang, et al., Plasma Sources Sci. Technol. **31** 115002 (2022)

Scaling networks (1/2)

- **Homogenous scaling relation**

$$G(x_1, t_1) = k^{\alpha[G]} G(x_k, t_k)$$



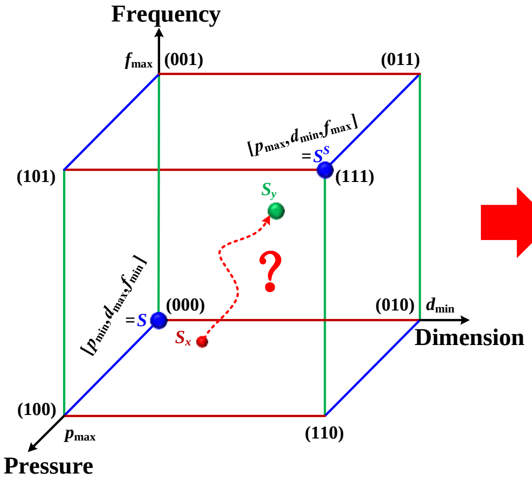


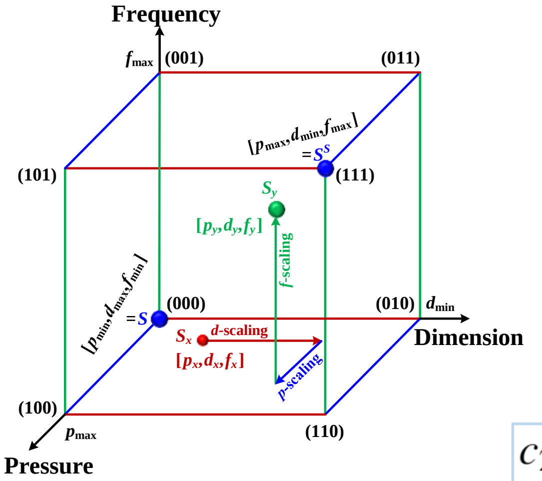
$$\Lambda_s = \sum_{k=0}^{n-1} C_n^k (n-k) = n \cdot 2^{n-1}$$

8 states yield 12 scaling relations (n=3)

Scaling networks (2/2)

- **Homogenous scaling relation**





$$\boxed{G(x_1, t_1) = k^{\alpha[G]} G(x_k, t_k)}$$

$$\boxed{G(k \cdot \mathbf{C}_k) = k^{\alpha[G]} G(\mathbf{C}_k)}$$

$$\mathbf{C}_k = [c_1, c_2, c_3] = [p_k^{-1}, d_k, f_k^{-1}]$$

$c_2/c_1 = pd$, $c_3/c_1 = f/p$, and $c_2/c_3 = fd$

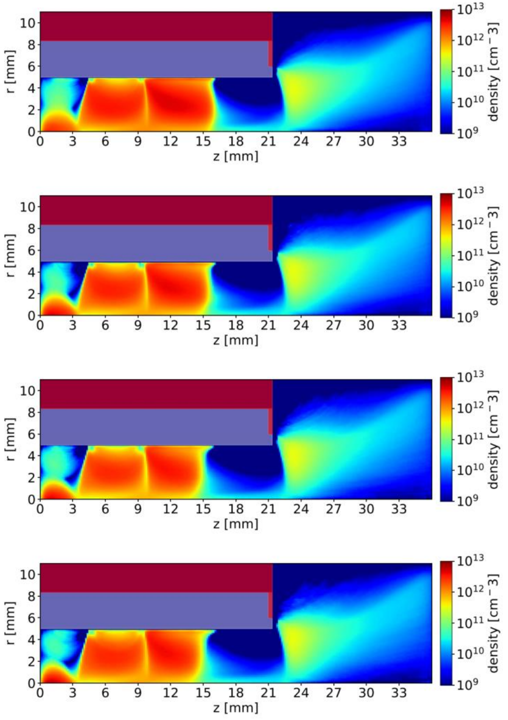
$$f(ax_1, ax_2, \cdots, ax_s) = a^n f(x_1, x_2, \cdots, x_s)$$

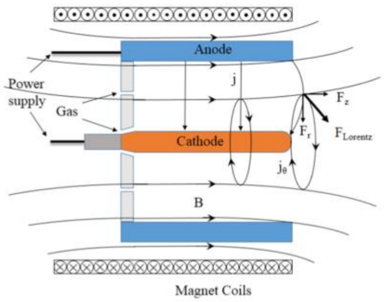
$$\Rightarrow \begin{cases} \sum_{\alpha=1}^{s} \frac{\partial f(x_1, x_2, \cdots, x_s)}{\partial x_\alpha} \cdot x_\alpha = nf(x_1, x_2, \cdots, x_s), \\ \sum_{\beta=1}^{s} \frac{\partial}{\partial x_\beta} \frac{\partial f}{\partial x_\alpha} \cdot x_\beta = (n-1) \frac{\partial f}{\partial x_\alpha}. \end{cases}$$

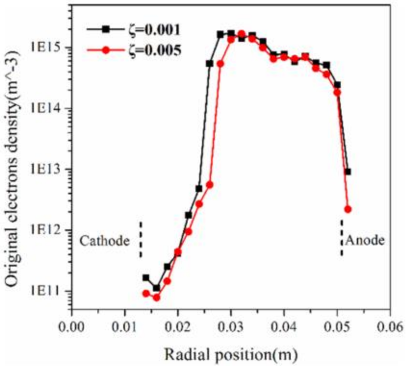
$$\boxed{\sum_j^n \frac{\partial G(\mathbf{C})}{\partial c_j} c_j = \alpha[G] \cdot G(\mathbf{C})} \Rightarrow \boxed{\sum_j^n \frac{\partial G(\mathbf{C})}{\partial c_j} c_j = \frac{\alpha[G]}{k^{\alpha[G]}} \cdot G(k \cdot \mathbf{C})}$$

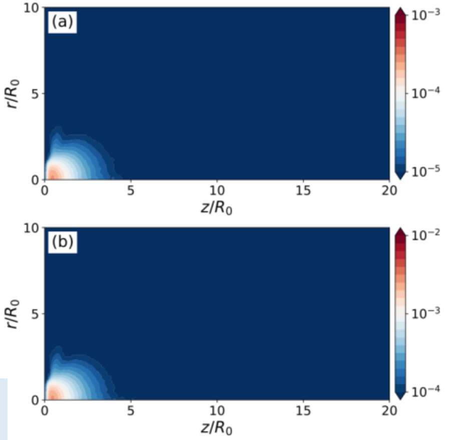
Other applications (1/3)

● Similarity and scaling for electric propulsion









Scaling methodology shows advantages in reducing the total computation of the plasma simulation [2]

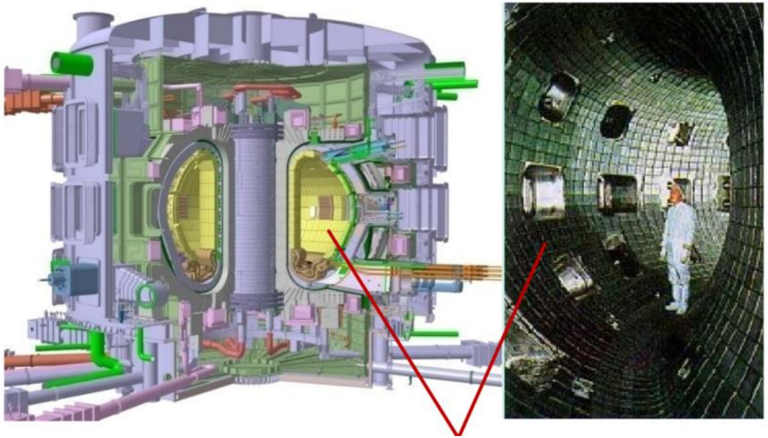
Similarity scaling-application to high-efficiency-multistage-plasma-thruster modelling (electron density distributions [1])

Geometrically self-similar ion acceleration in collisionless plasma beam expansion (ion density contour [3])

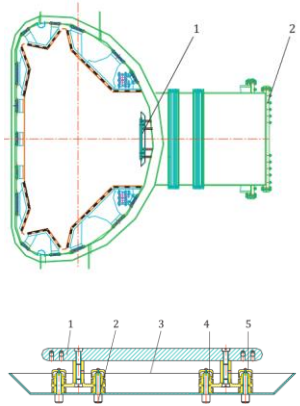
[1] P. Matthias, et al. Contrib. Plasma Phys. e201900199 (2020)
[2] J. Li, et al. J. Phys. D: Appl. Phys. **52** 455203 (2019)
[3] Y. Hu, et al. Plasma Sources Sci. Technol. **29** 125004 (2020)

Other applications (2/3)

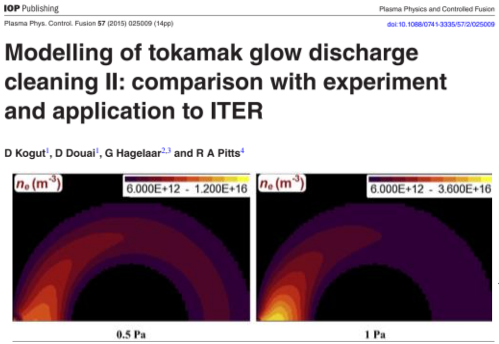
- **ITER GDC: Glow discharge cleaning**



ITER toroidal chamber for nuclear fusion



GDC electrode



1 Pa

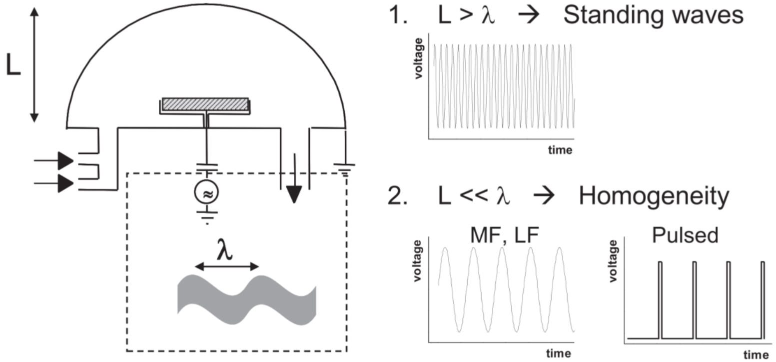
$p{\cdot}d = const$. For the laboratory tests, a scaling factor $a = 5$ has been used, corresponding to the ratio of the ITER minor radius ($d_{\mathrm{ITER}} = 2\,\mathrm{m}$) to the test chamber radius ($d_{\mathrm{TEST}} = 0.4\,\mathrm{m}$). Hence $p_{\mathrm{TEST}}{\cdot}d_{\mathrm{TEST}} \approx p_{\mathrm{ITER}}{\cdot}d_{\mathrm{ITER}}$, so that $p_{\mathrm{TEST}} = 1\,\mathrm{Pa}$ used in the experiments corresponds to $p_{\mathrm{ITER}} = 0.2\,\mathrm{Pa}$. Given the scaling factor,

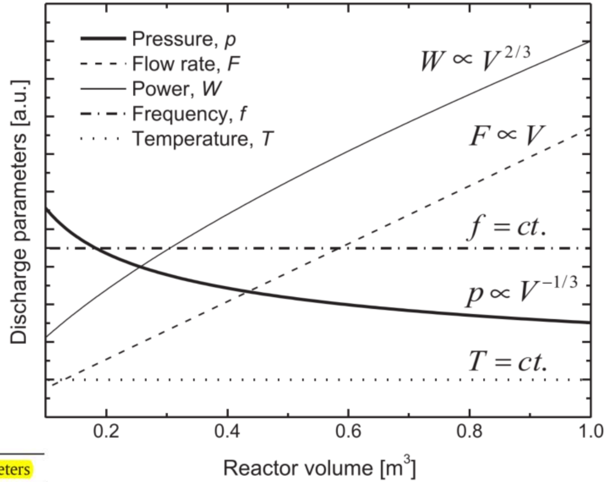
From toroidal to cylinder chamber

[1] D. Kogut, G. Hagelaar, et al. Plasma Phys. Control. Fusion (2015)
[2] M. Shimada et al, J. Nucl. Mater. (2011)
[3] J. Li et al. J. Nucl. Mater. (2011)

Other applications (3/3)

- **Upscaling plasma deposition**





Parameters	Processes Fig. 1	Issues	Solutions	Monitoring	Similarity parameters
Pressure	1,2,3,7	Interelectrode gap	Paschen law	Vacuum gauge	Kn
		Pressure gradients	Gas inlet distribution	QMS	s/λ_i
Residence time	2,7	Inactive zones	Stronger pumps	Pressure evolution	Da
		Dust + contamination	Gas inlet distribution	OES	Pe
Power density	2,3,5	Plasma edges	Sample arrangement	Langmuir + IV probe	W/F
(V_{sh}, I/A)		Arcing	Pulsed power	RFEA	n_e/F
Frequency	2,3,5	Standing waves	Lower frequency	Oscilloscope	L/λ
		Plasma position	Electrode shaping		
		Ion bombardment	Dual frequency		
Substrate temperature	4,5,6	Non-uniformity	Distributed heaters	Pyrometer	SZD
			Cooling down	Thermocouple	

Acronyms and variables: QMS: quadrupole mass spectrometry; OES: optical emission spectroscopy; RFEA: retarding field energy analyzer; SZD: structure zone diagram; V_{sh}: sheath voltage; I: electric current; A: electrode area; Kn: Knudsen number; s: sheath thickness; λ_i: ion mean free path; Da: Damköhler number; Pe: Péclet number; W: input power; F: gas flow rate, n_e: electron density; L: characteristic reactor size; λ: excitation wavelength.

C. Corbella, Surface & Coatings Technology 242, 237–245 (2014)

$$\frac{R_m}{F} = G\exp\left(-\frac{E_a}{W/F}\right)$$

Yasuda et al. (1976, 1977)
Rutscher and Pfau (1976)
Rutscher and Wagner (1985)
Hegemann et al. (2009, 2010)
A. Von Keudell, J. Benedikt (2010)

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Outline

1. **Introduction**
2. **Historical development**
3. **Mathematical derivation**
4. **Similarity in discharge plasmas**
   Breakdown, Transient, and Steady-state & Scaling networks & Other applications
5. **Summary**

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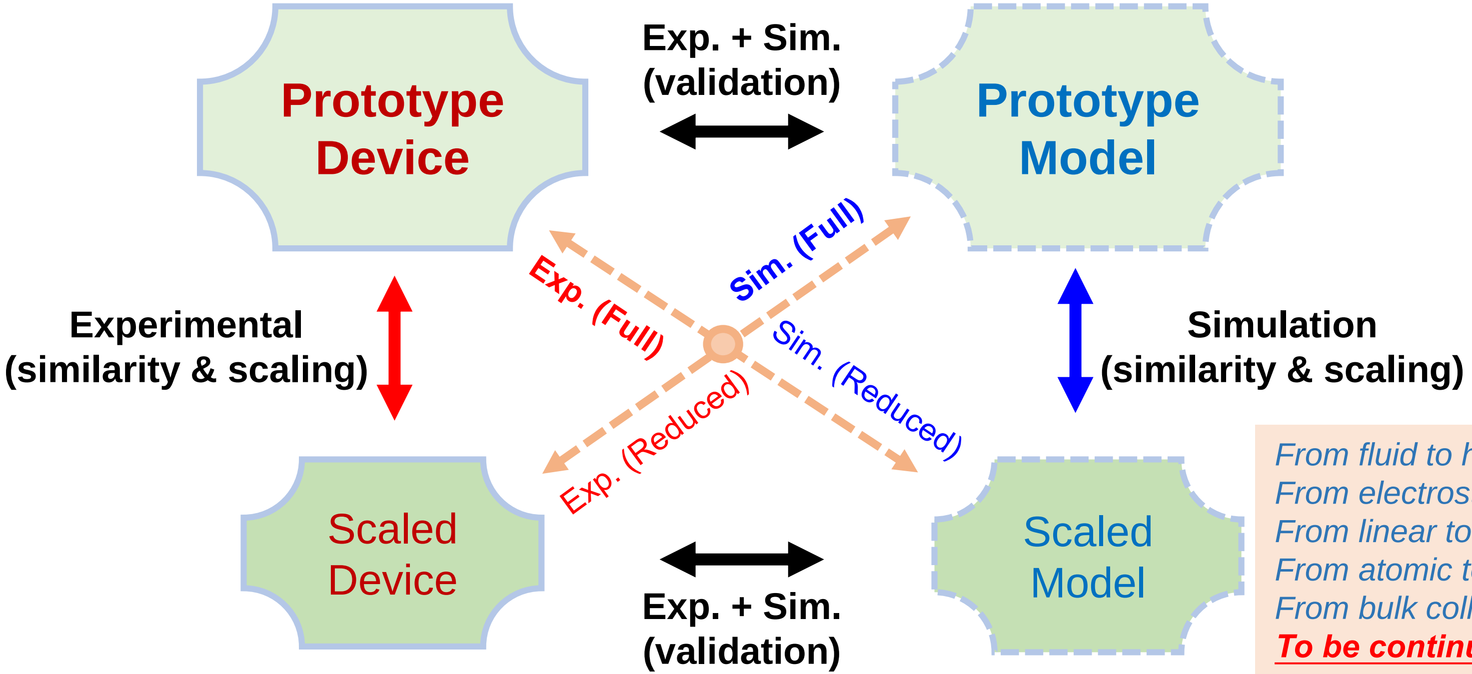
Summary

- ***How do physical laws remain the same when control parameters are changed?***



From fluid to hybrid and kinetic regimes;
From electrostatic to electromagnetic;
From linear to nonlinear mechanisms;
From atomic to molecular or gas mixtures;
From bulk collisions to surface processes.
To be continued…

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Thank you very much for your attention!

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